

Math 571: Model Theory
Spring Semester 2008
Prof. Ward Henson

Problem Set 1

Due in class Monday, February 11

There are four problems (equally weighted) and you should do all of them. To earn full credit requires a careful writeup of each problem, taking care to justify everything you claim and to explain your ideas clearly. Do not look up solutions in textbooks or other sources, and make sure your submitted solutions are your own work.

1.1 Problem. Let L be the language whose only nonlogical symbol is a binary predicate symbol E . Let \mathbb{K} be the class of L -structures in which E is interpreted by an equivalence relation that has precisely one equivalence class of cardinality n for each positive integer n . Let T be the theory of \mathbb{K} .

- Show that \mathbb{K} is the class of all models of T .
- Show that T does not admit QE.
- In which cardinalities κ is T κ -categorical? (T is κ -categorical iff T has a model of cardinality κ and any two models of T that have cardinality κ are isomorphic.)

1.2 Problem. Let L be the language whose only nonlogical symbols are a binary function symbol $+$ and a constant symbol 0 . For each field F let $V_2(F)$ be the L -structure whose underlying set is $\{(a, b) \mid a, b \in F\}$, whose interpretation of $+$ is given by $(a, b) + (a', b') = (a + a', b + b')$, for any $a, b, a', b' \in F$, and whose interpretation of 0 is $(0, 0)$. Let \mathbb{K} be the class of all L -structures that are isomorphic to $V_2(F)$ for some field F .

- Show that \mathbb{K} is closed under the ultraproduct construction.

1.3 Problem. Let L be the language whose only nonlogical symbol is the unary predicate symbol P . Let \mathbb{K} be the class of all L -structures \mathcal{A} such that $P^{\mathcal{A}}$ and $A \setminus P^{\mathcal{A}}$ are both infinite. Let T be the theory of the class \mathbb{K} . It is easy to show that $\mathbb{K} = \text{Mod}(T)$ and you may use this without proof.

- Classify the models of T up to isomorphism.
- Use local isomorphisms to show that T admits QE and is complete. (See the proof of Example 3.15.)

1.4 Problem. Consider the theory DLO of dense linear orderings without end points. Let $\mathcal{A} = (A, <)$ be a model of DLO .

- Characterize the functions $f: A \rightarrow A$ that are A -definable in \mathcal{A} .

(A function $f: A \rightarrow A$ is A -definable in \mathcal{A} if there exists a formula $\varphi(x, y, z_1, \dots, z_n)$ in the language of DLO and elements e_1, \dots, e_n of A such that $b = f(a) \iff \mathcal{A} \models \varphi[a, b, e_1, \dots, e_n]$ for all $a, b \in A$. By Example 3.15, φ can be taken to be a quantifier-free formula.)