Math 571: Model Theory
Spring Semester 2008
Prof. Ward Henson

Problem Set 2
Due in class Monday, February 25

There are four problems (equally weighted) and you should do all of them. To earn full credit requires a careful writeup of each problem, taking care to justify everything you claim and to explain your ideas clearly. Do not look up solutions in textbooks or other sources, and make sure your submitted solutions are your own work.

2.1 Problem. Let $\mathcal{A}, \mathcal{B}$ be L-structures such that $\mathcal{A} \equiv \mathcal{B}$.
• By example, show that there need not exist a local isomorphism from $\mathcal{A}$ onto $\mathcal{B}$. (This can be done with $\mathcal{A}$ being $(\mathbb{N}, n)_{n \in \mathbb{N}}$ or $(\mathbb{N}, <)$.)
• If both $\mathcal{A}$ and $\mathcal{B}$ are ω-saturated, show that there does exist a local isomorphism from $\mathcal{A}$ onto $\mathcal{B}$.

2.2 Problem. Let L be the language whose nonlogical symbols consist of a unary function symbol F. Let $\mathbb{K}$ be the class of all L-structures (A, f) in which f is a bijection from A onto itself and f has no finite cycles. It is easy to show there is an L-theory T such that $\mathbb{K} = \mathrm{Mod}(T)$, and you may use this fact without proof.
• Show that T admits QE and is complete.
(To say that a bijection $f \colon A \to A$ has no finite cycles is to say that $f^n(a) \neq a$ for each $n \geq 1$ and each $a \in A$. It follows that for each $a \in A$, the map from $\mathbb{Z}$ to A defined by $n \mapsto f^n(a)$ is 1-1.)

2.3 Problem. Let T and $\mathbb{K}$ be as in the preceding problem.
• Show that T has a countable ω-saturated model; describe it as explicitly as you can.
(You may use Problem 2.2 here, even if you did not solve it.)

2.4 Problem. Let L be the language whose nonlogical symbols are a binary predicate symbol P and a unary predicate symbol U. Let $\mathcal{F}$ be the set of all finite subsets of $\mathbb{N}$; we regard $\mathbb{N}$ and $\mathcal{F}$ as being disjoint. Let $\mathcal{A}$ be the L-structure with $A = \mathbb{N} \cup \mathcal{F}$, with $P^{\mathcal{A}} = \{(n, F) \in \mathbb{N} \times \mathcal{F} \mid n \in F\}$, and with $U^{\mathcal{A}} = \mathbb{N}$. Let κ be any infinite cardinal.
• Show that any κ^+-saturated L-structure that is elementarily equivalent to $\mathcal{A}$ must have cardinality at least 2^κ.