Math 571: Model Theory
Spring Semester 2008
Prof. Ward Henson

Problem Set 6
Due 5:00 pm, Monday, April 28

There are four problems (equally weighted) and you should do all of them. To earn full credit requires a careful writeup of each problem, taking care to justify everything you claim and to explain your ideas clearly. Do not look up solutions in textbooks or other sources, and make sure your submitted solutions are your own work.

6.1 Problem. Let T be a complete theory with infinite models and suppose $\mathcal{A}$ is an ω-saturated model of T. Let $\varphi(x,y)$ be an L-formula, with $x = x_1,\dots,x_m$ and $y = y_1,\dots,y_n$. Finally, let $a \in A^n$ and suppose $\varphi(x,a)$ is satisfiable in $(\mathcal{A},a)$.
• Show that there exists a type $p(x) \in S_m(A)$ that contains $\varphi(x,a)$ and satisfies $RM(p(x)) = RM(\varphi(x,a))$.
• Suppose $\varphi(x,a)$ is ranked. Show that the number of distinct types $p(x) \in S_m(A)$ that contain $\varphi(x,a)$ and satisfy $RM(p(x)) = RM(\varphi(x,a))$ is exactly $dM(\varphi(x,a))$. (In particular, this number is finite.)

6.2 Problem. Let T be a complete theory that has infinite models and let $\varphi(x)$ be the formula $x = x$, with x a single variable.
• Show that T is strongly minimal iff $RM(\varphi(x)) = dM(\varphi(x)) = 1$.

6.3 Problem. Let $\varphi(x,y)$ be a formula in the language of the theory DLO, with x a single variable and $y = y_1,\dots,y_n$. Let $\mathcal{A}$ be a model of DLO and $a \in A^n$.
• Determine the possible values of $RM(\varphi(x,a))$.
(Recall that DLO is the theory of dense linear orderings without end points; see Example 3.15.)

6.4 Problem. Let L be the language whose nonlogical symbols are countably many unary predicates $(P_n \mid n \in \mathbb{N})$. Let T be the L-theory axiomatized by the sentences expressing that $P_0 \supseteq P_1 \supseteq P_2 \supseteq \dots$, and that the complement of P_0 and each $P_n \setminus P_{n+1}$ are infinite.
• Using an approach like that used in solving Problems 4.1 and 5.2, show that T admits QE and is complete.
• Let $\mathcal{A}$ be any model of T and consider any subset S of A. Show that S is A-definable in $\mathcal{A}$ if and only if S is in the Boolean algebra of subsets of A generated by the finite subsets of A and the sets $P_n^{\mathcal{A}}$ for $n \in \mathbb{N}$.
• Calculate the Morley rank and degree of the formula $x = x$ in models of T. (Here x is a single variable.)