

UIUC Mock Putnam Exam (Elementary Version)  
1/2003

10-08-03

**Problem 1.** How many positive integers are there whose binary expansion has *exactly* 2003 digits and the sum of its binary digits is even.

**Problem 2.** Can 2003 be expressed as a sum of the squares of two integers?

**Problem 3.** If  $n$  is a positive integer, count the solutions in nonnegative integers  $x_1, \dots, x_k$  to the equation  $x_1 + \dots + x_k \leq n$ .

**Problem 4.** In how many ways can one distribute 100 slices of pizza between 20 students if every student gets at least one slice?

**Problem 5.** Evaluate the integral

$$I = \int_0^{\pi/2} \frac{dx}{1 + (\tan x)^{\sqrt{2}}}.$$