

UIUC Department of Mathematics

Mock Putnam Exam 1

October 5, 1998

This exam is intended as a practice test for the real Putnam Exam and will be graded in the same way. To receive credit, you need to explain yourself clearly and succinctly; an answer alone won't do.

Graded exams will be returned at next Monday's Putnam Training Session.

Solutions will be posted by the end of this week at
<http://www.math.uiuc.edu/hildebr/putnam/mockputnam.html>.

1. Find the smallest positive integer n such that every digit of $15n$ is either 8 or 0.
2. Water is poured from a cylindrical container of radius 1" and height 1" by slowly tilting the container to one side. What is the volume of water remaining when exactly half of the base of the container still is emerged in water?
3. Suppose that for each integer $n \geq 2$ we have one square S_n of size $1/\sqrt{n} \times 1/\sqrt{n}$. Show that for any number $\epsilon > 0$, we can "tile" a 1×1 square with elements of the collection $\{S_n\}$ such that the uncovered region of the 1×1 square has area less than ϵ . (No overlaps are allowed and all squares have to lie within the 1×1 square.)
4. Let $a, b, c > 0$ and $abc = 1$. What are the largest and smallest possible values of

$$S = \frac{1}{1+a+ab} + \frac{1}{1+b+bc} + \frac{1}{1+c+ca} ?$$

5. Let

$$I_\alpha = \int_0^\infty \frac{dx}{x^\alpha(1+x)}, \quad 0 < \alpha < 1.$$

Find the choice of α that minimizes I_α . Explain.