

UIUC Department of Mathematics

Mock Putnam Exam 1

September 27, 1999

This exam is intended as a practice test for the real Putnam Exam and will be graded in the same way. To receive credit, you need to explain yourself clearly and succinctly; an answer alone won't do.

Graded exams will be returned at next Monday's Putnam Training Session.

Solutions will be posted by the end of this week at
<http://www.math.uiuc.edu/~hildebr/putnam/mockputnam.html>.

1. Show that $\sqrt{17 - 12\sqrt{2}} + \sqrt{17 + 12\sqrt{2}}$ is an integer.

2. Evaluate the infinite product

$$(1 + 2^{-1})(1 + 2^{-2})(1 + 2^{-4})(1 + 2^{-8}) \dots$$

3. Show that there exists a multiple of 1999 that involves all 10 decimal digits.

4. Let $b(n)$ denote the number of binary strings of length n which do not contain the substring 11. For example, $b(3) = 5$ since there are 5 such strings of length 3: 000, 001, 010, 100, 101. Express $b(n)$ in terms of familiar sequences.

5. (Putnam, 1964) Let S be a set of $n \geq 1$ elements, and suppose A_1, A_2, \dots, A_k are distinct subsets of S such that (i) any two of these subsets intersect, and (ii) any subset of S that intersects each of the sets A_1, A_2, \dots, A_k must be one of the sets A_i . Show that $k = 2^{n-1}$.