UIUC Department of Mathematics

Mock Putnam Exam 2

October 11, 1999

This exam is intended as a practice test for the real Putnam Exam and will be graded in the same way. To receive credit, you need to explain yourself clearly and succinctly; an answer alone won't do.

Graded exams will be returned at next Monday's Putnam Training Session.

Solutions will be posted by the end of this week at
http://www.math.uiuc.edu/~hildebr/putnam/mockputnam.html.

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Problem 1. Let $x_1, x_2, \ldots, x_n$ be the numbers $1, 2, \ldots, n$, written in some order. Prove that

$$x_1x_2 + x_2x_3 + \cdots + x_{n-1}x_n + x_nx_1 \le 1^2 + 2^2 + \cdots + n^2.$$

Problem 2. (Putnam 1990) Prove or disprove that $\sqrt{2}$ is the limit of a sequence of numbers of the form $n^{1/3} - m^{1/3}$, where $n, m = 0, 1, 2, \ldots$

Problem 3. Prove that in the decimal expansion of $\sqrt{2}$ there is at least one non-zero digit between the millionth and the three-millionth decimal digits (inclusive) after the decimal point.

Problem 4. Let $s(n)$ denote the number of ways to write n as a sum of positive odd integers, with the order taken into account. For example, $s(5) = 5$ since 5 can be represented as $1+1+1+1+1$, $1+1+3$, $1+3+1$, $3+1+1$, and 5. Find a formula for $s(n)$.

Problem 5. Suppose 1000 points in the plane are given. Prove that there exists a circle such that there are exactly 500 points in the interior of the circle, and no point on the circle itself.