**Math 213, Section B1, Quiz 2 (SOLUTIONS); Friday, February 2, 2007**

**1.** Using induction, prove that for every integer $n \geq 2$

$$3^n \geq 2n^2 + 1.$$

**Solution. 1) Base of Induction.** First, check if the statement $3^n \geq 2n^2 + 1$ holds for $n = 2$.

We have $3^2 = 9$ and $2 \cdot 2^2 + 1 = 2 \cdot 4 + 1 = 9$. Since $9 \geq 9$, the required statement does hold for $n = 2$.

**2) Inductive Step.**

Let $k \geq 2$ and suppose that $3^k \geq 2k^2 + 1$ is known to hold. We need to derive that $3^{k+1} \geq 2(k+1)^2 + 1$, that is, $3^{k+1} \geq 2(k^2 + 2k + 1) + 1$, that is, $3^{k+1} \geq 2k^2 + 4k + 3$.

The inductive hypothesis

$$3^k \geq 2k^2 + 1 \text{ by multiplying by 3 implies}$$

$$3^{k+1} \geq 6k^2 + 3.$$

To show that $3^{k+1} \geq 2k^2 + 4k + 3$ it suffices to establish that for $k \geq 2$ we have $6k^2 + 3 \geq 2k^2 + 4k + 3$.

We have:

$$6k^2 + 3 \geq 2k^2 + 4k + 3 \text{ is equivalent to:}$$

$$4k^2 \geq 4k \text{ by dividing by } 4k > 0, \text{ is equivalent to:}$$

$$k \geq 1,$$

which holds since by assumption $k \geq 2$. Thus for $k \geq 2$ we have $6k^2 + 3 \geq 2k^2 + 4k + 3$, as required.