

Math 213, Quiz 11 (Solutions); Friday, April 25, 2008

PRINT YOUR NAME:

1.

(a) Let G be a simple connected graph with 12 vertices and 31 edges. Prove that G is non-planar.

(b) If G is a simple connected graph with 35 vertices, drawn in the plane so that G divides the plane into 20 regions, how many edges does G have? Justify your answer.

Solution.

(a) If G is a planar connected simple graph with $v \geq 3$ vertices and e edges then $e \leq 3v - 6$ By Corollary 1 from Ch. 9.7.

In this case $v = 12$ and $e = 31$ but

$$31 \not\leq 3 \cdot 12 - 6 = 30.$$

Therefore G is not planar.

(b) By Euler's formula, we have

$$r = e - v + 2$$

and hence $e = r + v - 2$. In this case $r = 20$, $v = 35$ and therefore the number of regions is:

$$e = r + v - 2 = 20 + 35 - 2 = 53.$$