Math 213, Section B1, Quiz 4 (Solution); Friday, February 8, 2008

1.
Prove that for every integer $n \geq 1$ we have

$$\sum_{j=1}^{n} \binom{n}{j} 5^j = 6^n - 1.$$

Solution.
By the Binomial Theorem for every $x, y \in \mathbb{R}$ we have

$$(x+y)^n = \sum_{j=0}^{n} \binom{n}{j} x^{n-j} y^j.$$

Substituting $x = 1$, $y = 5$ in the above formula, we get:

$$6^n = (1+5)^n = \sum_{j=0}^{n} \binom{n}{j} 1^{n-j} 5^j = \sum_{j=0}^{n} \binom{n}{j} 5^j =$$

$$= \binom{n}{0} 5^0 + \sum_{j=1}^{n} \binom{n}{j} 5^j = 1 + \sum_{j=1}^{n} \binom{n}{j} 5^j.$$

Therefore

$$\sum_{j=1}^{n} \binom{n}{j} 5^j = 6^n - 1,$$

as required.