SAMPLE MULTIPLE CHOICE QUESTIONS FOR THE 1-ST MIDTERM.

Choose the correct answer among the options provided.

(a) Let $f(x,y)$ be a function differentiable everywhere on $\mathbb{E}^2$ and suppose (x_0,y_0) is a relative minimum of f. Let $a = \frac{\partial f}{\partial x}(x_0,y_0)$ and $b = \frac{\partial f}{\partial y}(x_0,y_0)$ Then:

(1) (x_0,y_0) is not a critical point of f.
(2) (x_0,y_0) is an absolute minimum of f on $\mathbb{E}^2$.
(3) $e^{a+b} = 1$
(4) None of the above

Answer:

(b) Let $F(x,y)$ be a function differentialble at $(1,2)$ and let $\frac{\partial F}{\partial x}(1,2) = 7$, $\frac{\partial F}{\partial y}(1,2) = -5$. Let $\mathbf{v}$ be a nonzero vector such that $\bigtriangledown_{\mathbf{v}} F|_{(1,2)} = 0$

Then:

(1) $\mathbf{v} = [7,-5]$
(2) $\mathbf{v}$ is colinear with $[5,7]$.
(3) $\mathbf{v}$ is colinear with $[1,2]$.
(4) None of the above

Answer:

(c) Let $f(x,y)$ be a continuous function at (x_0,y_0).

Then:

(1)
$$\lim_{x\to x_0, y\to y_0} f(x,y) \text{ does not exist.}$$
(2)
$$\text{It is possible that } \lim_{x\to x_0, y\to y_0} f(x,y) \text{ does not exist.}$$
(3)
$$\lim_{x\to x_0, y\to y_0} f(x,y) \neq f(x_0,y_0)$$
(4) None of the above

Answer:

(d) Let $f = (f_1,\ldots,f_n) : \mathbb{E}^n \longrightarrow \mathbb{E}^n$ be a one-to-one differentiable function. Let $g = f^{-1}$ be the inverse function $g = (g_1,\ldots,g_n) : \mathbb{E}^n \longrightarrow \mathbb{E}^n$ which is also differentiable on $\mathbb{E}^n$. Suppose also that $f(\mathbf{O}) = \mathbf{O}$ where $\mathbf{O} = (0,\ldots,0)$.

Then:

(1) The Jacobian determinant $J_g(\mathbf{O}) = 0$.
(2) The Jacobian determinant $J_f(\mathbf{O}) = 0$.
(3) $J_g(\mathbf{O})J_f(\mathbf{O}) \neq 1$.
(4) $\frac{\partial f_1}{\partial x_1}|_{\mathbf{O}} \frac{\partial g_1}{\partial x_1}|_{\mathbf{O}} + \frac{\partial f_1}{\partial x_2}|_{\mathbf{O}} \frac{\partial g_2}{\partial x_1}|_{\mathbf{O}} + \ldots \frac{\partial f_1}{\partial x_n}|_{\mathbf{O}} \frac{\partial g_n}{\partial x_1}|_{\mathbf{O}} = 1$.

Answer:

(e) Let the curve C be given as an intersection of two surface:

$$F(x,y,z) = 0, G(x,y,z) = 0.$$

Let P be a point on this curve such that $\bigtriangledown F|_P = [3,5,0]$ and $\bigtriangledown G|_P = [1,1,0]$. Let $\mathbf{v}$ be a nonzero tangent vector to C at P.

Then:

(1) $\mathbf{v}$ is colinear with $[3, 5, 0]$.
(2) $\mathbf{v}$ is colinear with $[1, 1, 0]$.
(3) $\mathbf{v}$ is colinear with $[0, 0, 1]$.
(4) None of the above

Answer: