SAMPLE MULTIPLE CHOICE QUESTIONS FOR THE FINAL EXAM.

The final exam will take place on Friday, May 4, 8:00-11:00 AM in our usual classroom. The exam will be cumulative and will include all the material covered during the course. Specifically, the sections covered are:

Ch. 2.1-2.10, 2.12-2.19; Ch. 3.1-3.6; Ch. 4.3-4.7, Ch. 4.9; Ch 5.1-5.12.

The exam will be structured similarly to the midterms: there will be one multiple choice theoretical problem with several parts and nine computational problems. Review problems are homework problems assigned for these chapters. You should expect the computational problems on the exam to be somewhat easier than homework problems and somewhat harder than the quiz problems. You are allowed to use TWO two-sided sheets of notes on the final exam.

Problem 1. GO OVER the sample review questions for the midterms as well as over the midterms themselves (this material is available on the course web-page).

Problem 2.

Choose the correct answer among the options provided.

(a) Let R be the set of points in the xy-plane between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$, that is $R = \{(x, y)|1 \leq x^2 + y^2 \leq 4\}$. Suppose $P(x, y), Q(x, y)$ are functions which are continuous and have continuous partial derivatives everywhere in the xy-plane except the point $(0, 0)$. Suppose also $\partial Q/\partial x = 3 + \partial P/\partial y$ for all $(x, y) \neq (0, 0)$. Then

$$\oint_{x^2+y^2=4} (Pdx + Qdy) + \oint_{x^2+y^2=1} (Pdx + Qdy)$$

(where the first curve is oriented counter-clockwise and the second curve is oriented clockwise) is

(1) equal to 0;
(2) equal to 3π;
(3) equal to 9π;
(4) None of the above

Answer: (3)

(b) Let $P(x,y), Q(x,y)$ be functions which are continuous and have continuous partial derivatives everywhere in the xy-plane except the point $(0,0)$. Suppose that $\partial Q/\partial x = \partial P/\partial y$ for all $(x,y) \neq (0,0)$. Suppose also that $\oint_{x^2+y^2=1} (Pdx+Qdy) = 15$ (where the curve is oriented counter-clockwise).

Then the integral

$$\oint_{2x^2+5y^2=1} (Pdx + Qdy)$$

(where the curve is oriented counter-clockwise)

(1) equals 0;
(2) equals 15.
(3) equals -15.
(4) None of the above

Answer: (2)

(c) Let C be the curve $x = \cos t, y = \sin t, z = t^2 e^{t^4}$, where $0 \leq t \leq 2\pi$. Then

$$\int_C \ln(x^2 + y^2)\, ds$$

is equal to

(1) 0;
(2) 1;
(3) 2π;
(4) does not exist.

Answer: (1)

(d) Let S be the parameterized surface $x = u^2 + v^2, y = u + v, z = u - v$, where (u,v) varies over the uv-plane.

Then

(1) the surface S is not orientable;
(2) the surface S is orientable;
(3) the surface S has finite area;
(4) None of the above.

Answer: (2)

(e) Let R be the ball $(x-1)^2+y^2+(z+3)^2 \leq 1$ in the xyz-space. Consider the orientation on the sphere $(x-1)^2+y^2+(z+3)^2 = 1$ given by the outer normal vector to R. Then

$$\iint_S e^{(y^3+z^4)}\, dy\, dz \,-\, e^{(x^2z^5)}\, dz\, dx \,+\, z\, dx\, dy$$

(1) is equal to 0;
(2) does not exist;
(3) is equal 1;
(4) None of the above

Answer: (4)

(f) Consider the vector field $\mathbf{u} = 2y\mathbf{i} - y\mathbf{k}$ in the xyz-space. Let C_r be the circle of radius r around the origin in the xy-plane, that is $C_r = \{(x,y,z)|x^2+y^2 = r^2, z = 0\}$. We orient each C_r in the counter-clockwise direction in the xy-plane. Let $\mathbf{T}$ be the unit tangent vector to C_r.

Then

$$\lim_{r\to 0} \frac{1}{\pi r^2} \int_{C_r} u_T ds$$

(1) is equal to -2;
(2) is equal to 2;
(3) is equal to 0;
(4) None of the above

Answer: (1)