

Math 280 Section C1 Quiz 10

March April 19, 2001

Problem 1.

Let S be the parametric surface given by $x = u^2 + v^2$, $y = u + v$, $z = u - v$, where (u, v) varies in the region $0 \leq u \leq 1, 0 \leq v \leq 2$ in the uv -plane.

Compute the following surface integral:

$$\iint_S dx dz,$$

(where S is given the standard orientation coming from the parameterization). **Solution.**

We have

$$\frac{\partial(x, z)}{\partial(u, v)} = \begin{vmatrix} 2u & 2v \\ 1 & -1 \end{vmatrix} = -2u - 2v.$$

Therefore

$$\begin{aligned} \iint_S dx dz &= \int_0^2 \left(\int_0^1 (-2u - 2v) du \right) dv = \int_0^2 [-u^2 - 2vu]_{u=0}^{u=1} dv = \\ &= \int_0^2 -1 - 2v dv = [-v - v^2]_0^2 = -2 - 4 = -6. \end{aligned}$$