Math 280 Section C1 Quiz 3

February 9, 2000

Problem 1.

Consider the functions $u = u(x, y, z)$, $v = v(x, y, z)$ and $w = w(x, y, z)$ defined as

$$\begin{cases} u = x^3 + y^5 \\ v = 2xy - 3yz = 0 \\ w = z^3 \end{cases}$$

Compute the Jacobian determinant of the inverse map:

$$\frac{\partial(x, y, z)}{\partial(u, v, w)}$$

Solution

First we compute the Jacobian determinant of the direct function:

$$\frac{\partial(u, v, w)}{\partial(x, y, z)} = \begin{vmatrix} 3x^2 & 5y^4 & 0 \\ 2y & 2x - 3z & -3y \\ 0 & 0 & 3z^2 \end{vmatrix} = 3z^3(6x^3 - 9xz - 10y^5).$$

By the Inverse Function Theorem we have:

$$\frac{\partial(x, y, z)}{\partial(u, v, w)} = \frac{1}{\frac{\partial(u,v,w)}{\partial(x,y,z)}} = \frac{1}{3z^3(6x^3 - 9xz - 10y^5)} = \frac{1}{18z^2x^3 - 27x^2z^3 - 30z^2y^5}.$$