

Math 280 Section C1 Quiz 9

March April 13, 2001

**Problem 1.**

Let  $C$  be the curve in space given by  $x(t) = \cos t$ ,  $y(t) = \sin t$ ,  $z(t) = t$ , where  $0 \leq t \leq 2\pi$ .

Compute the following line integral:

$$\int_C (xy + z) ds$$

**Solution.**

We have

$$ds = \sqrt{(x')^2 + (y')^2 + (z')^2} dt = \sqrt{(-\sin t)^2 + \cos^2 t + 1^2} dt = \sqrt{2} dt.$$

Therefore

$$\begin{aligned} \int_C (xy + z) ds &= \int_0^{2\pi} (\cos t \sin t + t) \sqrt{2} dt = \sqrt{2} \int_0^{2\pi} (\cos t \sin t + t) dt = \\ &= \sqrt{2} \int_0^{2\pi} \left( \frac{1}{2} \sin(2t) + t \right) dt = \sqrt{2} \left[ -\frac{1}{4} \cos(2t) + \frac{t^2}{2} \right]_0^{2\pi} = 2\sqrt{2}\pi^2. \end{aligned}$$