Quiz 3 (Solution) March 4, 2004

Problem 1.

Find an equation $ay^{(3)} + by'' + cy' + dy = 0$, where $a, b, c, d \in \mathbb{R}$, $a \neq 0$, such that the function

$$y = -e^x + 3e^{-x}\cos(2x)$$

is a solution.

Solution.

The given function y is of the form $y = c_1e^x + c_2e^{-x}\cos(2x) + c_3e^{-x}\sin(2x)$ (with $c_1 = -1$, $c_2 = 3$, $c_3 = 0$). We need to find an equation $ay^{(3)} + by'' + cy' + dy = 0$ such that $r_1 = 1$ and $r_{2,3} = -1 \pm 2i$ are roots of its characteristic equation $ar^3 + br^2 + cr + d = 0$

Hence the characteristic equation itself should be:

$$(r-1)(r+1-2i)(r+1-2i) = 0,$$
$$(r-1)((r+1)^2 - (2i)^2) = 0, \quad (r-1)((r+1)^2 + 4) = 0,$$
$$(r-1)(r^2 + 2r + 5) = 0, \quad r^3 + 2r^2 + 5r - r^2 - 2r - 5 = 0,$$
$$r^3 + r^2 + 3r - 5 = 0.$$

The corresponding differential equation is:

$$y^{(3)} + y'' + 3y' - 5y = 0.$$