

**Problem 1.**

Consider the following wave equation problem

$$\begin{cases} y_{tt} = 9y_{xx}, & 0 < x < 2, t > 0 \\ y(0, t) = y(2, t) = 0, & t > 0 \\ y(x, 0) = 4x + 5, & 0 < x < 2, \\ y_t(x, 0) = 0, & 0 < x < 2. \end{cases}$$

Find the precise numerical value (NOT an infinite sum expression!) for  $y(0.5, 13)$ .

**Solution.**

We have  $a = \sqrt{9} = 3$  and  $L = 2$ .

We will use D'Alembert's form of the solution for this problem:

$$y(x, t) = \frac{1}{2}[f_O(x + at) + f_O(x - at)]$$

where  $f_O(x)$  is the 4-periodic odd extension of  $f(x) = 3x + 5$   $0 < x < 2$ .

Thus

$$\begin{aligned} y(0.5, 13) &= \frac{1}{2}[f_O(0.5 + 3 \cdot 13) + f_O(0.5 - 3 \cdot 13)] = \\ &= \frac{1}{2}[f_O(39.5) + f_O(-38.5)] = \frac{1}{2}[f_O(-0.5) + f_O(1.5)] = \\ &= \frac{1}{2}[-f(0.5) + f(1.5)] = \frac{1}{2}[-7 + 11] = 2. \end{aligned}$$