Quiz 2 (Solution) September 9, 2005

PRINT YOUR NAME:

Problem 1.

Find the general solution of the following differential equation on the interval $x > 0$:

$$\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{2y}{x} = \frac{2}{x^3 + x}.$$

Solution.

This is a linear first-order differential equation. First, we compute the integrating factor:

$$\rho(x) = e^{\int \frac{2}{x}\,\mathrm{d}x} = e^{2\ln|x|} = e^{\ln(x^2)} = x^2.$$

We now multiply the original equation by $\rho(x) = x^2$ and get:

$$\frac{\mathrm{d}y}{\mathrm{d}x}x^2 + 2yx = \frac{2x^2}{x^3 + x} = \frac{2x}{x^2 + 1}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}(yx^2) = \frac{2x}{x^2 + 1}$$

$$yx^2 = \int \frac{2x}{x^2 + 1}\,\mathrm{d}x = \int \frac{\mathrm{d}(x^2 + 1)}{x^2 + 1} = \ln|x^2 + 1| + C = \ln(x^2 + 1) + C$$

$$y = \frac{\ln(x^2 + 1)}{x^2} + \frac{C}{x^2}.$$

Thus the general solution of our equation on the interval $x > 0$ is

$$y = \frac{\ln(x^2 + 1)}{x^2} + \frac{C}{x^2}$$

where $C \in \mathbb{R}$ is an arbitrary constant.