Quiz 4 (Solutions)   September 23, 2005

Problem 1.

Consider the following differential equation on $(-\infty, \infty)$:

$$(*) \qquad y'' + 2y' - 3y = 0$$

(1) Find the general solution of equation (*) on $(-\infty, \infty)$;

(2) Solve the initial value problem

$$y'' + 2y' - 3y = 0, \quad y(0) = 2, \quad y'(0) = -5$$

on $(-\infty, \infty)$.

Solution.

(1) The characteristic equation of this 2-nd order homogeneous differential equation with constant coefficients is:

$$\lambda^2 + 2\lambda - 3 = 0, \quad (\lambda + 3)(\lambda - 1) = 0$$

and it has two distinct real roots $\lambda_1 = -3$ and $\lambda_2 = 1$. Therefore the general solution of our differential equation on $\mathbb{R}$ is

$$y = c_1 e^{-3x} + c_2 e^x \quad \text{where } c_1, c_2 \in \mathbb{R} \text{ are arbitrary constants.}$$

(2) From (1) we know that this initial value problem has the solution of the form $y = c_1 e^{-3x} + c_2 e^x$ for some $c_1, c_2 \in \mathbb{R}$. Hence $y' = -3c_1 e^{-3x} + c_2 e^x$.

The conditions $y(0) = 2$ and $y'(0) = -5$ give us

$$c_1 e^{-0} + c_2 e^0 = 2, \quad c_1 + c_2 = 2$$

and

$$-3c_1 e^{-30} + c_2 e^0 = -5, \quad -3c_1 + c_2 = 5.$$

Solving the system

$$c_1 + c_2 = 2, -3c_1 + c_2 = 5$$

we get $c_1 = 7/4$ and $c_2 = 1/4$.

Therefore

$$y = \frac{7}{4} e^{-3x} + \frac{1}{4} e^x.$$