

Quiz 11 (Solution); Friday, April 29, 2005

For each of the following subsets determine whether it is an ideal in the given ring. If not, explain why not.

1. $\{f(x) \in \mathbb{Z}[x] : f(0) = 0\} \subseteq \mathbb{Z}[x]$;
2. $\{a + bi | a, b \in \mathbb{Q}\} \subseteq \mathbb{C}$;
3. $\{[0]_6, [3]_6\} \subseteq \mathbb{Z}/6\mathbb{Z}$.

Solution.

(1) This is an ideal in $\mathbb{Z}[x]$. In fact, one can check that it is a principal ideal:

$$\{f(x) \in \mathbb{Z}[x] : f(0) = 0\} = (x) \triangleleft \mathbb{Z}[x];$$

(2) The set $I = \{a + bi | a, b \in \mathbb{Q}\}$ is not an ideal in \mathbb{C} .
Indeed, $1 = 1 + 0 \cdot i \in I$ but $\sqrt{2} \cdot 1 = \sqrt{2} \notin I$.

(3) Yes, this is an ideal in $\mathbb{Z}/6\mathbb{Z}$ and in fact, this is a principal ideal:

$$\{[0]_6, [3]_6\} = ([3]_6) \triangleleft \mathbb{Z}/6\mathbb{Z}.$$