

Quiz 8 (Solutions); Friday, April 8, 2005

Find the number of elements of S_6 which commute with the permutation

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 3 & 2 & 4 & 5 & 1 \end{pmatrix}.$$

Solution.

After decomposing σ as a product of disjoint cycles we get $\sigma = (1\ 6)(2\ 3)$.

Thus the conjugacy class σ^{S_6} consists of all permutations in S_6 whose cycle structure is the product of two disjoint transpositions. The number of such permutations is

$$|\sigma^{S_6}| = \binom{6}{2} \binom{4}{2} \cdot \frac{1}{2} = 45.$$

By Corollary 2.143

$$[S_6 : C_{S_6}(\sigma)] = |\sigma^{S_6}| = 45.$$

Therefore the number of elements commuting with σ in S_6 is

$$|C_{S_6}(\sigma)| = \frac{|S_6|}{[S_6 : C_{S_6}(\sigma)]} = \frac{6!}{45} = 16.$$