Quiz 9 (Solutions); Friday, April 15, 2005

For each of the following determine whether it is a commutative ring. If not, explain why not.

1. $(S, +, \cdot)$, where

$$S = \{\frac{m}{2^n} : m, n \in \mathbb{Z}, \text{ and } n \geq 0\},$$

   and where $+, \cdot$ are the standard addition and multiplication.

2. $(M(2, \mathbb{R}), +, \cdot)$ where $M(2, \mathbb{R})$ is the set of all 2×2 matrices with real entries and where $+, \cdot$ are the standard matrix addition and multiplication.

3. $(\mathbb{R}_{>0}, +, \cdot)$ where $+, \cdot$ are the standard addition and multiplication.

Solution.

(1) S is a ring since it is a subring of $\mathbb{Q}$.

(2) This is not a ring since there exist matricies $A, B \in M(2, \mathbb{R})$ such that $AB \neq BA$.

(3) This is not a ring since $(\mathbb{R}_{>0}, +)$ is not an abelian group: it has no neutral elemment since $0 \notin \mathbb{R}_{>0}$ and no additive inverses.