

Extra Credit Problem Set 1; Due Friday, March 7.

Print your name:

1.

Let G be a group and let $S \subseteq G$.

Define

$$\langle S \rangle := \cap \{H : H \leq G, S \subseteq H\}$$

(a) Prove that $\langle S \rangle \leq G$.

(b) Prove that

$$\langle S \rangle = \{s_1^{\epsilon_1} \dots s_n^{\epsilon_n} : n \geq 0, s_i \in S, \epsilon_i = \pm 1\}.$$

Here if $n = 0$ the product $s_1^{\epsilon_1} \dots s_n^{\epsilon_n}$ is interpreted as e .

For $S \subseteq G$ we call $\langle S \rangle$ *the subgroup generated by S* . A subgroup $H \leq G$ is said to be *finitely generated* if there exists a finite S such that $H = \langle S \rangle$.

2. For the group $(\mathbb{Q}, +)$ prove that every finitely generated subgroup is cyclic.

3. Prove that the groups $(\mathbb{Q}, +)$ and $(\mathbb{Q}^2, +)$ are not isomorphic.