Extra Credit Problem Set 5; Due Friday, April 25.

Print your name:

**1.**

Let $R$ be a commutative ring with 1. Let $I$ be an ideal in $R[x]$ such that the lowest degree of a nonzero polynomial in $I$ is $n \geq 1$ and such that $R$ contains a monic (that is, with leadding coefficient 1) polynomial of degree $n$. Prove that $I$ is a principal ideal.

**2.** Prove that for every $n \geq 1$ the ring $M_n(\mathbb{R})$ has no proper ideals. That is, prove that if $I \triangleleft M_2(\mathbb{R})$ then either $I = 0$ or $I = M_2(\mathbb{R})$.

**Hint.** It is enough to prove that if $A \in M_n(\mathbb{R})$ is a nonzero matrix then the principal ideal $\langle A \rangle$ is equal to $M_n(\mathbb{R})$. Think about the effect of multiplying on (the left or on the right) a matrix in $M_n(\mathbb{R})$ by an elementary matrix $E_{k,l}$ (where $(E_{k,l})_{i,j} = 1$ if $k = i, j = l$ and $(E_{k,l})_{i,j} = 0$ otherwise).