MA. 481 #3 DUE: THURS, FEB 21 NAME: ______________

$O(n)$ = "the orthogonal group"

= all choices of orthonormal basis $\{\vec{e}_1, \dots, \vec{e}_n\}$ of $\mathbb{R}^n$

= all $n \times n$ real matrices A such that $AA^T = I$

writing $A = \begin{pmatrix} -\vec{e}_1- \\ \vdots \\ -\vec{e}_n- \end{pmatrix}$

$$\begin{pmatrix} x^1 & x^2 & x^3 & x^4 \\ y_1 & y_2 & y_3 & y_4 \\ z & z & z & z \\ w^1 & w^2 & w^3 & w^4 \end{pmatrix}$$

1 a) $O(4)$ is the solution set of

______ equations in the variables $(x^1, x^2, x^3, x^4, y^1, \dots, z^1, \dots, w^1 \dots$

Write these equations.

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b) Write the ___ $\times 16$ Jacobian matrix Dg:

c) Show $Dg(A)$ has full rank (ie, its rows are linearly independent) for every $A \in O(4)$. So by IFT, $O(4)$ is a manifold. (There is a really-easy way, and a not-too-hard way.)

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d) By IFT, any $A \in O(4)$ is in a coordinate patch whose coordinates are chosen from $(x^1, \ldots, w^4)$. Give a choice that will work for $A = I$. Explain.

2) The same argument works to show $O(n)$ is a manifold. What is the dimension of $O(n)$? Justify this 2 ways:
a) by the first definition of $O(n)$ above.

b) by the IFT.