H/wk 8, due March 28 NAME:

1. Let ω be a 2-form on $\mathbb{R}^3$ given by
$$\omega = 2xy\, dx\wedge dy - (x^2+y+1)\, dy\wedge dz$$
a) find $d\omega$

b) Let $X = 2xy\frac{\partial}{\partial x} + (z^2+x^2)\frac{\partial}{\partial z}$.
Compute the 1-form $i_X(\omega)$

c) Let $f: \underset{(u,v)}{\mathbb{R}^2} \longrightarrow \underset{(x,y,z)}{\mathbb{R}^3}$ be given by
$$f(u,v) = (u+v,\ u^2+1,\ 3uv-4)$$
Compute the form $f^*\omega$ on $\mathbb{R}^2$

d) Let $\eta = dx + dy + 3dz$. Compute $\omega\wedge\eta$.

2. Consider the following vector fields on $\mathbb{R}^3$:
$$X = 2yx\frac{\partial}{\partial x} + (z^2+x^2)\frac{\partial}{\partial z}$$
$$Y = e^{xy}\frac{\partial}{\partial x} + \cos(2xz)\frac{\partial}{\partial y}$$
Compute the vector field $[X,Y]$ on $\mathbb{R}^3$

3. Let X, Y, Z be smooth vector fields on M. Prove the Jacobi Identity:

$$[[X,Y],Z] + [[Y,Z],X] + [[Z,X],Y] = 0$$

4. Let ω be an r-form on a vector space V, where $r \geq 2$. Let $v_1, v_2, .., v_r \in V$ be linearly dependent vectors. Prove that $\omega(v_1, v_2, .., v_r) = 0$

5. let $\omega = \alpha(x,y,z)\, dx \wedge dy + \beta(x,y,z)\, dx \wedge dz + \gamma(x,y,z)\, dy \wedge dz$

a) Write an explicit condition for α, β, γ equivalent to the condition $d\omega = 0$.

b) Write an explicit condition for α, β, γ equivalent to saying that ω is exact.

c) Prove that

$$\omega = z^2\, dx \wedge dy + 2zy\, dx \wedge dz$$

is an exact 2-form on $\mathbb{R}^3$.