1. Differential equations and power series

2. Radius of convergence

(1) $\sum_{k=0}^{\infty} \frac{x^{3k}}{k!}$,
(2) $\sum_k x^{2^k}$,
(3) $\sum_k \frac{x^k}{k^2!}$,
(4) $\sum_k \frac{x^{k^2}}{k!}$.
(5) $\sum_k (\ln x)^k$,

3. Differential equations

Features: Find the equation for coefficients, solve them and calculate the radius of convergence.

(1) $y' + 2y'' = 3y$, $y(0) = y'(0) = 1$.
(2) $2y' + y'' = 3y$, $y(0) = y'(0) = 1$.
(3) $2y' + y'' = 3y$. Find all the real numbers a such that

$$y = \sum_k \frac{a^k}{k!} x^k$$

is a solution.

(4) $y' + xy = y''$. $y(0) = 1 = y'(0)$.
  a) Find the equation for the coefficients $y(x) = \sum_k a_k x^k$.
  b) Calculate a_6.
  c) Show that $|a_{k-1}| \leq 1$ and $|a_{k+1}| \leq 1$ implies $|a_{k+1}| \leq 1$.
  d) Assume that $|a_k| \leq 1$ for all k. What can you say about the radius of convergence? Is it true that for the solution y above we have

$$\int_0^x y(t)dt = \sum_k a_k \frac{x^{k+1}}{k+1} \ ?$$

(5) $y' = 2 + y^2$, $y(0) = 0$. Find $y^{44}(0)$
  a) By the power series method. (Hint $a_0 = 0$, $a_2 = 0$, $a_4 = 0$,....)
  b) By solving the equation (separation of variables).