

Math 361, Section X1, Fall 2002
Exam 1, October 2

SHOW ALL WORK TO QUALIFY FOR FULL CREDIT.

Maximum possible score: 100 Points

1. 25 points Consider a box containing 6 white balls and 8 black balls. Pick the balls from the box one by one.
 - (a) 10 points What is the probability that the fourth ball is white and the fifth ball is black?
 - (b) 10 points What is the probability that the fourth ball is white?
 - (c) 5 points What is the probability that the fifth ball is black given that the fourth ball is white?

Hint: you may want to “reserve” balls.

2. 15 points An instructor gives her class 15 problems, telling them that the exam will consist of a random selection of 7 of them. By the time of the exam, a student knows how to do 12 out of the 15.
 - (a) 8 points What is the probability that the student knows how to do all of the questions on the exam?
 - (b) 7 points What is the probability that the student knows how to do at least 6 of the questions on the exam?

Hint: compare this problem to a lottery problem.

3. 15 points There are 7 restaurants in a town. Suppose three students are hungry, and each student randomly chooses a restaurant.
 - (a) 7 points What is the probability that no two students go to the same restaurant?
 - (b) 8 points What is the probability that exactly two students go to the same restaurant?
4. 30 points Suppose that each child born to Jim and Jane is equally likely to be a boy or girl, independently of the gender of all other children. Jim and Jane have 5 children. Compute the following probabilities:
 - (a) 10 points All children are of the same sex.
 - (b) 5 points Exactly 3 are boys
 - (c) 10 points youngest and oldest are girls
 - (d) 5 points At least one boy.

5. 15 points Suppose that

$$\mathbb{P}(A \cap C|B) = 0.4 \quad \mathbb{P}(B) = 0.1 \quad \mathbb{P}(A \cap B \setminus C) = 0.02 \quad \text{and} \quad \mathbb{P}(A \setminus B) = 0.2$$

Compute

(a) 5 points $\mathbb{P}(A \cap B)$

(b) 5 points $\mathbb{P}(A)$

(c) 5 points $\mathbb{P}(A \cup B)$

ANSWERS

1. (a) Reserve a black ball and a white ball.

$$\frac{8 \times 6 \times (12)_3}{(14)_5}$$

- (b) Reserve a white ball.

$$\frac{6 \times (13)_3}{(14)_4}$$

- (c) Note that

$$\frac{6 \times (13)_3}{(14)_4} = \frac{6 \times (13)_4}{(14)_5}.$$

Answer is

$$\frac{8 \times 6 \times (12)_3}{6 \times (13)_4} = \frac{8}{13}.$$

2. Define $q \stackrel{\text{def}}{=} 1/\binom{15}{7}$.

(a) $\binom{12}{7}q$.

(b) $((\binom{12}{7} + \binom{12}{6}\binom{3}{1}))q$.

3. Define $q \stackrel{\text{def}}{=} 1/7^3$.

(a) $\binom{7}{3}3!q$.

(b) $\binom{3}{2}\binom{7}{2}2!q$.

4. Set $q \stackrel{\text{def}}{=} (1/2)^5$.

(a) $2q$

(b) $\binom{5}{3}q$

(c) 2^3q

(d) $1 - q$.

5. (a)

$$\begin{aligned} \mathbb{P}(A \cap B) &= \mathbb{P}(A \cap B \cap C) + \mathbb{P}(A \cap B \setminus C) \\ &= \mathbb{P}(A \cap C|B)\mathbb{P}(B) + \mathbb{P}(A \cap B \setminus C) = 0.4 \times 0.1 + 0.02 = 0.06. \end{aligned}$$

(b) $\mathbb{P}(A) = \mathbb{P}(A \cap B) + \mathbb{P}(A \setminus B) = 0.06 + 0.2 = 0.26$.

(c) $\mathbb{P}(A \cup B) = \mathbb{P}(B) + \mathbb{P}(A \setminus B) = 0.1 + 0.2 = 0.3$.