

1. 10 points Consider the matrix

$$A \stackrel{\text{def}}{=} \begin{pmatrix} 5 & 3 \\ -1 & 1 \end{pmatrix}$$

Find the eigenvalues and associated eigenvectors.

ANSWERS

1. Characteristic equation is

$$0 = \det \begin{pmatrix} \lambda - 5 & -3 \\ 1 & \lambda - 1 \end{pmatrix} = \lambda^2 - 6\lambda + 8 = (\lambda - 2)(\lambda - 4);$$

thus the eigenvalues of A are $\lambda_1 = 2$ and $\lambda_2 = 4$. For λ_1 , solve the equation

$$\begin{pmatrix} -3 & -3 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix};$$

take $\mathbf{e}_1 \stackrel{\text{def}}{=} (1, -1)^T$. For λ_2 , solve the equation

$$\begin{pmatrix} -1 & -3 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix};$$

take $\mathbf{e}_2 \stackrel{\text{def}}{=} (-3, 1)^T$.