

1. We want to find solutions of the ODE

$$\ddot{x}(t) + \dot{x}(t) - 6x(t) = 5e^t$$

- (a) Find the roots of the characteristic equation for this ODE.
- (b) Find a particular solution of this ODE
- (c) Find the general solution.

ANSWERS

1. (a) Characteristic equation is $0 = (\lambda^2 + \lambda - 6) = (\lambda + 3)(\lambda - 2)$; roots are $\lambda_1 = -3$ and $\lambda_2 = 2$.

(b) Assume $y_p(t) = \alpha e^t$; then

$$5e^t = \ddot{y}_p(t) + \dot{y}_p(t) - 6y_p(t) = \alpha(1 + 1 - 6)e^t = -4\alpha e^t;$$

set $\alpha = -5/4$;

$$y_p(t) = -\frac{5}{4}e^t$$

(c) General solution is

$$y_p(t) = -\frac{5}{4}e^t + C_1e^{-3t} + C_2e^{2t}.$$