

1. Consider the function

$$f(t) \stackrel{\text{def}}{=} \begin{cases} 1 & \text{if } 0 \leq t \leq 1/2 \\ 0 & \text{if } 1/2 < t < 1 \end{cases}$$

We want to write

$$f(t) = \sum_{n=-\infty}^{\infty} c_n \exp[2\pi nit].$$

Find the c_n 's.

ANSWERS

1. In general,

$$c_n = \int_{t=0}^1 f(t) \exp[-2\pi nit] dt = \int_{t=0}^{1/2} \exp[-2\pi nit] dt.$$

Thus $c_0 = 1/2$ and for $n \neq 0$,

$$c_n = \frac{1 - \exp[-\pi ni]}{2\pi in} = \begin{cases} 0 & \text{if } n \text{ is even} \\ \frac{1}{\pi ni} & \text{if } n \text{ is odd.} \end{cases}$$