Math 451, Spring 2004
Homework 1, due January 30

1. **[10 points]** Fix a set Ω. A *monotome class* $\mathscr{M}$ is a collection of subsets of Ω which is closed under monotone limits. In other words,

   - if $A_1 \subset A_2 \subset A_3 \dots$ where the A_i's are in $\mathscr{M}$, then $\cup_{i=1}^{\infty} A_i \in \mathscr{M}$
   - if $A_1 \supset A_2 \supset A_3 \dots$ where the A_i's are in $\mathscr{M}$, then $\cap_{i=1}^{\infty} A_i \in \mathscr{M}$.

   (a) Fix a collection $\mathscr{C}$ of subsets of Ω. Show that

   $$M(\mathscr{C}) \stackrel{\text{def}}{=} \bigcap \{\mathscr{M} : \mathscr{C} \subset \mathscr{M} \text{ and } \mathscr{M} \text{ is a monotone class}\}$$

   is the smallest monotone class containing $\mathscr{C}$.

   (b) Fix a field $\mathscr{F}$ of subsets of Ω. Show that $\sigma(\mathscr{F}) = M(\mathscr{F})$.

2. **[10 points]** Fix a probability triple $(\Omega, \mathscr{F}, \mathbb{P})$.

   (a) Show that if $A_1 \subset A_2 \subset A_3 \dots$ where the A_i's are in $\mathscr{F}$, then $\lim_{n \nearrow \infty} \mathbb{P}(A_n) = \mathbb{P}(A)$, where $A \stackrel{\text{def}}{=} \cup_{n=1}^{\infty} A_n$.

   (b) Show that if $A_1 \supset A_2 \supset A_3 \dots$ where the A_i's are in $\mathscr{F}$, then $\lim_{n \nearrow \infty} \mathbb{P}(A_n) = \mathbb{P}(A)$, where $A \stackrel{\text{def}}{=} \cap_{n=1}^{\infty} A_n$.

3. **[10 points]** Exercise 1.17

4. **[10 points]** Exercise 1.18