

1. 10 points Fix a cumulative distribution function F . Let U be a random variable which is uniform on $(0, 1)$. Define

$$G(t) \stackrel{\text{def}}{=} \inf\{s \in \mathbb{R} : F(s) > t\}$$

for all $t \in \mathbb{R}$ (where $\inf \emptyset = \infty$), and then define $Y \stackrel{\text{def}}{=} G(U)$. Show that the distribution of Y is F (Hint: you may want to first consider points of continuity of F).

2. 10 points Let X be a real-valued random variable, and define $\sigma\{X\} \stackrel{\text{def}}{=} X^{-1}(\mathcal{B}(\mathbb{R}))$. Show that $\sigma\{X\}$, thus defined, is a sigma-algebra.