

1. 15 points Consider a solution of the recursion relation $h_n = ph_{n+1} + qh_{n-1}$, where $p \in (0, 1)$ and $q = 1 - p$. Find a recursion for the column vector $(h_{n+1}, h_n)^T$.
2. 15 points Consider a Markov chain on $I = \{0, 1, 2, \dots, N\}$. Fix $p \in (0, 1)$ and set $q \stackrel{\text{def}}{=} 1 - p$. Assume $p_{i,i+1} = p$ for $i \in \{0, 1, 2, \dots, N-1\}$ and $p_{i,i-1} = q$ for $i \in \{1, 2, \dots, N-1\}$. Assume $p_{N,N} = 1$. Compute $\mathbb{P}_i\{H^A < \infty\}$, where $A \stackrel{\text{def}}{=} \{0\}$.
3. 15 points Exercise 1.3.1
4. 15 points Exercise 1.3.2
5. 15 points Exercise 1.3.3.