Math 564, Fall 2005 **Homework 4**
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1. 15 points 1.6.1

2. 15 points 1.6.2

3. 45 points Let's now consider *sampling* of a Markov chain. Assume that X is a Markov chain with transition matrix P. Let $\{\xi_n\}_{n=1}^{\infty}$ be an i.i.d. collection of geometric random variables with parameter q. By geometric random variables we mean that

$$\mathbb{P}\{\xi_n = k\} = q(1-q)^k$$

for all $k \in \{0, 1 \dots\}$. We assume that the ξ_n's are independent of X. Set $S_0 \overset{\text{def}}{=} 0$ and recursively define

$$S_{n+1} \overset{\text{def}}{=} S_n + \xi_{n+1}$$

for all $n \in \{0, 1 \dots\}$.

Define now

$$Y_n \overset{\text{def}}{=} X_{S_n}$$

for all $n \in \{0, 1 \dots\}$; i.e., we sample X in geometric increments of time.

   (a) 15 points Show that Y is Markov.

   (b) 15 points Compute the transition matrix for Y. NOTE: Take the matrix perspective.

   (c) 15 points Compute the distribution of the first time that $Y_n \neq Y_0$.