Math 595, Fall 2005 **Midterm**

Richard Sowers **due April 4**

Fix a Polish space X and $\mu \in \mathscr{P}(\mathsf{X})$.

1. 10 points Show that μ is *regular*; i.e., that $\mu = \mu_* = \mu^*$, where

$$\mu_*(A) \stackrel{\text{def}}{=} \sup\{\mu(F) : F \subset A \text{ and } F \text{ closed}\}$$
$$\mu^*(A) \stackrel{\text{def}}{=} \inf\{\mu(G) : G \supset A \text{ and } G \text{ open}\}$$

for all $A \in \mathscr{B}(\mathsf{X})$. Hint: the collection of sets A for which $\mu(A) = \mu^*(A) = \mu_*(A)$ is a _ _ _ _ _ - _ _ _ _ _ _ _ _ .

2. 10 points Show that μ is itself tight. Hint: the fact that X is separable means . . . .

3. 10 points Fix $\nu \in \mathscr{P}(\mathsf{X})$ such that $H(\nu|\mu) < \infty$. Suppose that $\{A_n;\, n \in \mathbb{N}\} \subset \mathscr{B}(\mathsf{X})$ is such that $\lim_{n\to\infty} \mu(A_n) = 0$. Show that $\lim_{n\to\infty} \nu(A_n) = 0$.