Malliavin calculus for product measures on $\mathbb{R}^{\mathbb{N}}$ based on chaos

Horst Osswald

Abstract

Malliavin calculus is being developed for measures μ on $\mathbb{R}^{\mathbb{N}}$, which are products of arbitrary Borel probability measures μ^1 on $\mathbb{R}$. Each μ-square integrable functional on $\mathbb{R}^{\mathbb{N}}$ can be expanded into an orthogonal series of multiple integrals. The integrators are martingales, whose increments are orthogonal polynomials of $\mathfrak{E}$, where $\mathfrak{E}$ is a Borel measurable bijection from $\mathbb{R}$ onto $\mathbb{R}$ such that $\int_{\mathbb{R}} e^{|\mathfrak{E}(x)|} d\mu^1(x) < \infty$. Based on this chaos decomposition result, we introduce the Malliavin derivative, the Itô integral, the Skorohod integral and prove the Clark Ocone formula, which plays an important role in finance mathematics. Our approach includes Malliavin calculus on the classical Poisson space and on any abstract Wiener Fréchet space over "little" l_2. Moreover, measures, for which polynomials are not integrable, and also non-smooth measures are included.

We will give some ideas, how the results can possibly be used to establish Malliavin calculus for Lévy processes.