Untying knots and Von Neumann traces

In the 1980's, Mike Freedman won a Field's medal by showing that four dimensional manifolds with the homotopy type of a sphere are homeomorphic to a sphere (the 4-dimensional Poincaré conjecture). This captures stunningly the type of classification result that topologists love. It turns the extremely difficult problem of recognizing a sphere into potentially computable algebra and homotopy theory.

In his central argument, Freedman finds sufficient criteria to embed a disk with a fixed boundary in a topological four manifold. This *disk embedding theorem* fundamentally underlies the study of four dimensional manifolds. Freedman uses delicate and remarkable infinite constructs along the way.

The knot-slice problem is a special case of the problem of finding embedded disks—one which also plays a central role in singularity theory and characteristic classes. In this talk I discuss how Freedman's seminal result and the knot-slice problem intimately intertwine, both through the toolbox uesd for study and the spirit of the arguments. I will introduce the knot-slice problem, discuss its history, explore the relation with Freedman's work, and discuss recent advances in the field.

Finally, I will introduce the newly uncovered role of analytic techniques derived through infinite dimensional unitary representations and Von Neumann signatures.

The talk will address a general mathematical audience.