SOME RECENT RESULTS IN W*-RIGIDITY

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Abstract. Two measure preserving actions of countable groups on probability spaces, $\Gamma \curvearrowright X, \Lambda \curvearrowright Y$, are *$W^*$-equivalent* if their associated von Neumann algebras $L^\infty(X) \rtimes \Gamma$, $L^\infty(Y) \rtimes \Lambda$ are isomorphic. This is weaker than *orbit equivalence* of actions, requiring an isomorphism of probability spaces $\Delta : X \simeq Y$ with $\Lambda\Delta(t) = \Delta(\Gamma t)$, $\forall t \in X$ a.e., which in turn is weaker than "classical" *conjugacy* of actions. Free ergodic measure preserving actions of amenable groups are undistinguishable under W*-equivalence (Connes '76), in fact even under orbit equivalence (Ornstein-Weiss '80). But a rich and deep rigidity theory underlies the non-amenable case. I will present some recent results in this direction, one of which shows that any isomorphism of von Neumann algebras associated with Bernoulli actions $\Gamma \curvearrowright [0,1]^\Gamma$, $\Lambda \curvearrowright [0,1]^\Lambda$, of Kazhdan groups Γ, Λ, comes from a conjugacy of the actions.