This talk concerns the application of relatively classical tools from real harmonic analysis (namely, the $T(1)$-theorem for spaces of homogenous type) to the novel context of several complex variables. Specifically, I will present recent joint work with E.M. Stein on the extension to higher dimension of Calderón’s and Coifman-McIntosh-Meyer’s seminal results about the Cauchy integral for a Lipschitz planar curve (interpreted as the boundary of a Lipschitz domain $D \subset \mathbb{C}$). From the point of view of complex analysis, a fundamental feature of the 1-dimensional Cauchy kernel $H(w, z) = \frac{1}{2\pi i} (w-z)^{-1} \, dw$ is that it is holomorphic as a function of $z \in D$. In great contrast with the one-dimensional theory, in higher dimension there is no obvious holomorphic analogue of $H(w, z)$. This is because geometric obstructions arise (the Levi problem), which in dimension one are irrelevant. A good candidate kernel for the higher dimensional setting was first identified by Jean Leray in the context of a $C^\infty$-smooth, convex domain $D$: while these conditions on $D$ can be relaxed a bit, if the domain is less than $C^2$-smooth (never mind Lipschitz!) Leray’s construction becomes conceptually problematic. In this talk I will present (i) the construction of the Cauchy-Leray kernel and (ii) the $L^p(bD)$-boundedness of the induced singular integral operator under the weakest currently known assumptions on the domain’s regularity—in the case of a planar domain these are akin to Lipschitz boundary, but in our higher-dimensional context the assumptions we make are in fact optimal. The proofs rely in a fundamental way on a suitably adapted version of the so-called "$T(1)$-theorem technique" from real harmonic analysis. Time permitting, I will describe applications of this work to complex function theory—specifically, to the Szegő and Bergman projections (that is, the orthogonal projections of $L^2$ onto, respectively, the Hardy and Bergman spaces of holomorphic functions).
The notion of conformal mapping is of fundamental importance in complex analysis. Conformal maps are used by mathematicians, physicists and engineers to change regions with complicated shapes into much simpler ones, and to do so in a way that preserves shape on a small scale (that is, when viewed up close). This makes it possible to ‘transpose’ a problem that was formulated for the complicated-looking region into another, related problem for the simpler region (where it can be easily solved)—then one uses conformal mapping to “translate” the solution of the problem over the simpler region, back to a solution of the original problem (over the complicated region). The beauty of conformal mapping is that its governing principle is based on a very simple idea that is easy to explain and to understand (much like the statement of Fermat’s celebrated last theorem).

In the first part of this talk I will introduce the notion of conformal mapping and will briefly go over its basic properties and some of its history (including a historical mystery going back to Galileo Galilei). I will then describe some of the many real-life applications of conformal maps, including: cartography; airplane wing design (transonic flow); art (in particular, the so-called ‘Droste effect’ in the work of M.C. Escher). Time permitting, I will conclude by highlighting a 2013 paper by MacArthur fellow L. Mahadevan that uses the related notion of quasi-conformal mapping to link D’Arcy Thompson’s iconic work On Shape and Growth (published in 1917) with modern morphometric analysis (a discipline in biology that studies, among other things, how living organisms evolve over time).

No previous knowledge of complex analysis is needed to enjoy this talk.