MATH 415; LESSON PLAN

WILLIAM J. HABOUSH

1. General Information

There will be two midterm exams and a final. The midterms are worth 100 points each and the final is worth 200 points. The total for homeworks is 80 points. Homework is assigned on tuesdays and thursdays but it is collected and returned on the thursday of the week following the week of its assignment. Professor Haboush will see students in his office (305 Altgeld hall) during office hours and by appointment. Office hours will be Tuesday from 1:15 to 2:15 and Wednesday from 4 to 5. In addition JiYoung Kim will answer questions on Monday from 2 to 4 PM in 1 Illini Hall and Qiang Zheng will hold office hours in the graduate student room in the basement of Coble Hall on Tuesday from 4 to 6 PM. Professor Haboush can be reached in his office at 333 6498. His email address is haboush@math.uiuc.edu Graduate students or honors students may receive additional credit for research projects. All projects must be discussed with Professor Haboush and they must receive his approval.

2. Text

The text is **Applied Linear Algebra** by Olver and Shakiban.

3. Assignments and Exams

. *Week 1, Jan. 20'th and 22'nd.*
  (1) *1/20: Section 1.1*
  (2) *1/22: 1.1.1, b,d,f; 1.2.5; 1.2.7*

. *Week 2, Jan. 27'th and 29'th Sections 1.3, 1.4 and 1.5*
  (1) *1/27: 1.2.1, a,c,f; 1.4.2, a,b,c; 1.4.3, a,c,d*
  (2) *1/29: 1.4.19, a,c,d; 1.5.1*

. *Week 3, Feb. 3'rd and 5'th. Sections 1.6, 1.7, 1.8*
  (1) *2/3: 1.5.32 a,c,d; 1.6.1 a,b,c,d; 1.6.2; 1.6.3*
  (2) *2/5: 1.7.9; 1.7.13; 1.7.16 a; 1.8.2 a,c,d,f; 1.8.7*

. *Week 4, Feb. 10'th and 12'th*

(1) *2/10: 1.8.22 a,b,e; 1.9.1 b,e,f; 1,9,2; 1.9.5 omit d; 2.3.1; 2.3.2*
(2) *2/12: 2.3.4 a,b,d; 2.3.21 a,c,e,f; 2.3.23; 2.3.28*

. *Week 5, Feb. 17'th and 19'th*
  (1) *2/17: 2.4.1 a,b,c; 2.4.2 a,c; 2.4.3; 2.4.8 a,b; 2.5.2 a,c,d; 2.5.5 b,d,f; 2.5.21 a,c; 2.5.23; 2.5.24 ii) vii)*
  (2) *2/19: 2.6.1 a,c; 2.6.3 a,c,d.*

. *Week 6, Feb. 24'th and 26'th*
  (1) *2/24; 3.1.2 a,d,e,f; 3.1.3; 3.1.7; 3.1.9; 3.1.19 a,b; 3.1.21 a,b; 3.1.22*
  (2) *2/26—*

. *Week 7, March 3'rd and 5'th*
  (1) *3/3: MIDTERM I*
  (2) *3/5: 3.4.22 i) iii), v); 3.4.26; 3.4.32; 3.5.1 c,d,e; 3.5.2 c,f; 3.5.19 a,c,e.*

. *Week 8, March 10'th and 12'th.*
  (1) *3/10: 4.1.2 a,c,e; 4.2.1; 4.2.3 a,b,c.*
  (2) *3/12: 4.3.1, 4.3.2; 4.3.5; 4.3.6 b; 4.3.9 a, b i), ii); 4.3.14 a,c; 4.3.15 a,c,d; 4.4.1; 4.4.3*

. *Week 9, March 17'th and 19*
  (1) *3/17: 5.1.1; 5.1.4; 5.1.13; 5.1.18; 5.1.21; 5.1.27*
  (2) *3/19: 5.2.1 5.2.6 a,b; 5.2.10 a; 5.2.17 a,c.*

. *Week 10, March 31'st and April 2'nd*
  (1) *3/31: 5.3.1; 5.3.7; 5.3.26; 5.3.27 a,d,f; 5.3.28 a, b i),ii)*
  (2) *4/2: 5.5.1 a,c,d,e; 5.5.3; 5.5.4; 5.5.11; 5.5.15 a,b; 5.6.1 a,b,c; 5.6.4 a,b; 5.6.17 a,c,e; 5.6.21 b,c*

. *Week 11, April 7'th and 9'th*
  (1) *4/7: 6.2.1 a,c; 6.2.2 a,b*
  (2) *4/9: 7.1.2; 7.1.7; 7.1.19 a,c,e,l,m; 7.1.27 a,d; 7.1.32; 7.1.33 a,d; 7.1.37 a-e,h,i*

. *Week 12, April 14'th and 16'th*
  (1) *4/14: MIDTERM II*
  (2) *4/16: 7.1.38; 7.1.51 a,b,e; 7.2.24 a,b,d; 7.2.25; 7.2.26*

. *Week 13, April 21'st and 23'rd*
  (1) *4/21: 8.2.1 a,b,e,g,i,j,k; 8.2.2; 8.2.3; 8.2.5; 8.2.6; 8.2.7; 8.2.10*
  (2) *4/23: 8.2.14; 8.2.16; 8.2.20; 8.3.31; 8.3.32; 8,2,35;*

- *Week 14, April 28'th and 30'th*
  1. *4/28: 8.3.1; 8.3.2 a,b,f,g,h,i; 8.3.3; 8.3.7; 8.3.8; 8.3.10*
  2. *4/20: 8.3.15 a,b,d,f,g; 8.3.18; 8.3.19; 8.4.1; 8.4.21*
- *Week 15, May 5'th Review and Questions and Answers.*

4. Course Syllabus

Math 415. Applied Linear Algebra Syllabus for Instructors Students in this course are primarily from engineering and related departments such as physics. Emphasis should be on explaining and relating basic concepts of linear algebra, and showing how they apply to, and arise in, concrete problems. In general, simple proofs should be included only as time permits, and outlining the key arguments of a proof in a special case often suffices. However, students do need to be able to do and understand manipulations in abstract vector spaces(such as testing for linear independence). Applications to be covered are in sections 2.6., 4.4., 5.5., 6.2., and chapter 9.

Note: The overall pace in this course will be demanding, so plan carefully.

Chapter 1. Linear Algebraic Systems (6 hours) Chapter 1 should be covered rapidly but carefully since all students should have some experience with linear systems and determinants from calculus. Gaussian elimination will be used throughoutthecourseto findthefourfundamental matrixsubspaces(introduced inchapter 2). Determinants will be crucial to determine eigenvalues and students should become familiar with their basic properties. 1.1. Solution of Linear Systems 1.2. Matrices and Vectors 1.3. Gaussian Elimination?Regular Case 1.4. Pivoting and Permutations 1.5. Matrix Inverses(omit Gauss-Jordan Elimination) 1.6. Transposes and Symmetric Matrices 1.8. General Linear Systems 1.9. Determinants

Chapter 2. Vector Spaces and Bases (6 hours) This chapter should lead to (and give a proof of) the fact that a vector space has a well-defined dimension (Theorem 2.29 in section 2.4). It also gives a first strong application of theconcepts justd eveloped(i.e. the fundamental matrix subspaces) to graphs and incidence matrices. The application section 2.6. sets the stage for electrical networks, which will be covered in section 6.2. 2.1. Real Vector Spaces 2.2. Subspaces 2.3. Span and Linear Independence 2.4. Bases and Dimension 2.5. The Fundamental Matrix Subspaces 2.6. Graphs and Incidence Matrices

Chapter 3. Inner Products and Norms (4 hours) Inner products other than the dot product are used in the applications in chapter 5 and 6 and are essential for the proof of the spectral theorem. If one plans to prove the spectral theorem via hermitian inner products, these should be introduced here (see comments on use of complex numbers in chapter 8 below). If the spectral theorem is to be proved using minimization of quadratic forms, then the Cauchy-Schwarz inequality is important, in particular that it continues to hold for positive semi-definite symmetric bilinear forms (without the ?strict inequality? part of the statement of Cauchy-Schwarz). 3.1. Inner Products 3.2. Inequalities 3.4. Positive Definite Matrices 3.5. Completing theSquare (first subsection only)

Chapter 4. Minimization and Least Squares Approximation (3 hours) The main sections are 4.3. (least squares and the closest point) and section 4.4., the application to data fitting. Least squares and their applications will be taken up again in greater depth in section 5.5. 4.1. Minimization Problems 4.2. Minimization of Quadratic Functions 4.3. Least Squares and the Closest Point 4.4. Data Fitting and Interpolation (first subsection only)

Chapter 5. Orthogonality (6 hours) The applications in the second part of section 5.5. (in particular example 5.42 and its introduction) provide a good test for the students to see if the abstract concepts (subspaces, span, orthogonality, non-standard inner products) are understood. The second part of section 5.6 explains the geometry of matrix multiplication, setting the stage for chapter 7 on linear functions. 5.1. Orthogonal Bases 5.2. The Gram-Schmidt Process (first subsection only) 5.3. Orthogonal Matrices (omit Householder?s Method) 5.5. Orthogonal Projections and Least Squares 5.6. Orthogonal Subspaces

Chapter 6. Equilibrium (2 hours) Many students are familiar with the physical laws governing electrical networks (i.e. Kirchhoff?s andOhm?slaws) but have not seen that these laws can be captured in terms of the geometry of matrix multiplication, using the incidence matrix of the underlying graph and the conductance matrix of the network. 6.2. Electrical Networks

Chapter 7. Linearity (3 hours) The main section is 7.2. Students should see the basic linear transformations (rotation, reflection, stretching, and shear), learn how to represent the linear transformation by a matrix after choosing a basis, and understand how the matrix representation depends on the choice of basis. This is studied in greater depth for diagonalizable matrices in chapter 8. 7.1. Linear Functions 7.2. Linear Transformations

Chapter 8. Eigenvalues (5 hours) Most students taking this course are already familiar with complex numbers. A brief review of how to manipulate them (addition, multiplication, conjugation, and modulus) and an appeal to the fundamental theorem of algebra suffice for the calculation of eigenvalues. A proof of the spectral theorem avoiding

Hermitian matrices and complex inner products can be build on the minimization characterization of the eigenvalues of a symmetric matrix. This is alluded to in the second part of section 8.4., but the argument is not given in the text. 8.1. Simple Dynamical Systems 8.2. Eigenvalues and Eigenvectors 8.3. Eigenvector Bases and Diagonalization 8.4. Eigenvalues of Symmetric Matrices

Chapter 9. Linear Dynamical Systems (4 hours) 9.1. Basic Solution Techniques 9.2. Stability of Linear Systems 9.3. Two-Dimensional Systems

Leeway and Examinations (4 hours