Spring 2006

Math 490. Topology of Surfaces

Instructor: Professor Jeremy Tyson

Is it possible to comb the hair on a round sphere so that all of the hair lies flat against the surface? If a closed wire loop is dipped in a bucket of soap and removed, the resulting soap film may take a variety of shapes; what types of surfaces can be obtained? How many colors are required to color all maps on the surface of a torus (doughnut) so that no two adjacent countries receive the same color?

Questions of this sort are the subject of *geometric topology*. This course will cover some basic ingredients of this subject, in one dimension (knot theory) and two dimensions (surface theory). As such it serves as both a complement and counterpoint to a course in classical point-set topology. The main goal of this course is to prove the fundamental classification theorem for compact surfaces (two-dimensional manifolds), which gives an explicit list of all such surfaces up to topological type. Along the way we will discuss numerous other topics and ideas arising within this broad area: the topological nature of vector fields in the plane and on surfaces, knot theory and surfaces spanned by knots and links, and algebraic invariants in topology (*homology*). If time permits, we will touch on some applications of these topics in the sciences (topology of DNA, shape of the universe, topological methods in robotics), and/or preview the geometric side of the theory (*differential geometry*).

This course should be of interest to students in mathematics and in a variety of scientific fields (computer science, physics, engineering, etc.), and, taken in conjunction with a course in point set topology, should provide a strong and solid foundation for graduate study in geometry and topology.

Prerequisites:

Must have completed the standard calculus sequence (through **Math 242/243**).

Linear algebra: **Math 415** preferred, **Math 225** probably also sufficient, but would require additional work from the student.

Math 347 or an equivalent course in another department, with a suitable level of mathematical rigor.

No prior knowledge or experience in topology is necessary; we will begin with a "crash course" in topology in the plane and abstract topological theory on surfaces sufficient for the material of this course.

Textbook:

Required: E. D. Bloch, *"A First Course in Geometric Topology and Differential Geometry"* (Birkhäuser, 1997)
Recommended: C. Adams, *"The Knot Book"* (Freeman and Co, 1994)

Coursework:

The course will have regular homework assignments, one or two midterm exams, and a final exam.

Need more information?
Contact Prof. Tyson at tyson@math.uiuc.edu or 244-4132.