

Course Outline —595, Topics: Advanced methods in Probabilistic Combinatorics— FALL 2006

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The Probabilistic Method is a powerful tool in tackling many problems in discrete mathematics. It belongs to those areas of mathematics which have experienced a most impressive growth in the past few decades.

This course provides an extensive treatment of the Probabilistic Method, with emphasis on methodology. We will try to illustrate the main ideas by showing the application of probabilistic reasoning to various combinatorial problems. The topics covered in the class will include (but are not limited to):

Linearity of expectation, the second moment method, the local lemma, correlation inequalities, martingales, large deviation inequalities, Janson and Talagrand inequalities, pseudo-randomness, random graphs, random regular graphs, Szemerédi Regularity Lemma, percolation, bootstrap percolation.

TEXTBOOKS: Most of the topics covered in the course appear in the books listed below (especially the first one). Other topics appear in recent papers.

The Probabilistic Method, by N. Alon and J. H. Spencer, 2nd Edition, Wiley, 2000.

Random Graphs, by B. Bollobás, 2nd Edition, Cambridge University Press, 2001.

Random Graphs, by S. Janson, T. Luczak and A. Rucinski, Wiley, 2000.

Graph Coloring and the Probabilistic Method, by M. Molloy and B. Reed, Springer, 2002.

COURSE OUTLINE: (The selection of topics will be from the list below, and additional results may be added from recent papers):

- Hypergraph 2-coloring, Sum-free subsets.
- The second moment method, distinct sums, random graphs and threshold functions, cliques in random graphs.
- Alteration method, graphs with high girth and high chromatic number, improved hypergraph 2-coloring, bounding of large deviations (Chernoff bounds).
- Lovasz Local Lemma (including proof) and its applications. Correlation inequalities.
- Martingales, chromatic number of random graphs
- Probabilistic gems: Independence number of triangle-free graph, crossing numbers, incidence and additive number theory
- Concept of random regular graphs.
- Szemerédi Regularity Lemma
- Percolation, bootstrap percolation.

REQUIREMENTS: There will be several homework assignment, in order that the students could check their understanding of the material. For a full credit some homework should be submitted or a research paper should be presented in the class. There are no exams.

RESOURCES: Electronic mail is a medium for announcements and questions.

PREREQUISITES: There are no official prerequisites, but students need the mathematical maturity and background for graduate-level mathematics. For example, basics of linear algebra, probability and graph theory are assumed to be known.