
1. Let \( f : M \to \mathbb{R} \) be a smooth function. What are the conditions on the differential of \( f \) that guarantee that \( N := f^{-1}(0) \) is an embedded submanifold? Are these conditions necessary?

Suppose \( N = f^{-1}(0) \) is a submanifold and \( X, Y \) are two vector fields on \( M \) that are tangent to \( N \). Prove that the Lie bracket \([X, Y]\) is tangent to \( N \) as well.

2. The set
   \[ M = \{(x, y, z, w) \in \mathbb{R}^4 \mid x = 8 - 2y^2 - 2z^2 - 2w^2, x \geq 0\} \]
   is a 3-manifold with boundary. Pick an orientation of \( M \) and compute the integral
   \[ \int_M d(xy) \wedge dz \wedge dw. \]

3. Let \( \nabla \) be a connection on a manifold \( M \). Prove that the torsion \( T \) of the connection defined by
   \[ T(X, Y) = \nabla_X Y - \nabla_Y X - [X, Y], \]
   where \( X, Y \) are vector fields, is actually a tensor on \( M \). What do you need to check?

4. Let \( f : M \to N \) be a submersion and \( g : P \to N \) a smooth map \((M, N, P)\) are manifolds). Is the set
   \[ \{(m, p) \in M \times P \mid f(m) = g(p)\} \]
a submanifold of \( M \times N \)? Explain.