1. Prove by definition that if $\mu_1, \ldots, \mu_n$ are measures on $(X, \mathcal{A})$, $\alpha_1, \ldots, \alpha_n \in [0, \infty)$, then
   \[ \sum_{j=1}^{n} \alpha_j \mu_j \text{ is a measure on } (X, \mathcal{A}). \]

2. Compute \( \lim_{k \to \infty} \int_0^k x^n (1 - k^{-1}x)^k \, dx \). Here \( n \in \mathbb{N} \).

3. Let \( \mu^* \) be an outer measure on \( X \). \( \{A_j\} \) be a sequence of disjoint \( \mu^* \)-measurable sets. Prove that \( \mu^*(E \cap \bigcup_j A_j) = \sum_j \mu^*(E \cap A_j) \).

4. Suppose that \( \{f_n\} \) is a sequence of positive measurable functions on \( \mathcal{R} \), \( \lim_{n \to \infty} f_n(x) = f(x) \) at every \( x \in \mathbb{R} \), and \( \int_{E} f = \lim_{n \to \infty} \int_{E} f_n < \infty \). Prove that \( \int_{E} f = \lim_{n \to \infty} \int_{E} f_n \) for all measurable sets \( E \).

5. Let \( E \) be a Lebesgue measurable set in \( \mathbb{R} \) and \( m(E) > 0 \). Prove that for any \( 1 > \varepsilon > 0 \), there is an open interval \( I \) such that \( m(E \cap I) > \varepsilon m(I) \).

6. Let \( f \in L^p(\mathbb{R}) \) with \( 1 \leq p < \infty \). Prove that
   \[ \lim_{\lambda \to 0} \lambda^p m(\{x \in \mathbb{R} : |f(x)| > \lambda\}) = \lim_{\lambda \to \infty} \lambda^p m(\{x \in \mathbb{R} : |f(x)| > \lambda\}) = 0 \]