1. Given a map $f : X \to X$, define $C_f := X \times [0, 1]/\sim$ where we identify $(x, 1) \sim (f(x), 0)$ for each $x \in X$.

   (a) Let $X = S^1 \subset \mathbb{C}$, and $f : X \to X$ by $f(z) = z$. Compute the homology of $C_f$.

   (b) Let $X = S^1 \subset \mathbb{C}$ and $f : X \to X$ by $f(z) = z^2$. Compute the homology of $C_f$.

2. Let $(X, x_0)$ and $(Y, y_0)$ be based spaces. Suppose both $x_0$ and $y_0$ admit neighborhoods for which there exist deformation retracts to $\{x_0\}$ and $\{y_0\}$ respectively. Let $V := X \amalg Y/\sim$ where we identify $x_0 \sim y_0$. Using only the Eilenberg-Steenrod axioms (Dimension, Sum, Exact sequence of a pair, Homotopy, Excision), compute $H_* V$ in terms of $H_* X$ and $H_* Y$.

3. Let $X \subseteq \mathbb{R}^2$ be the subset

   $$X := S^1 \cup (\mathbb{R} \times \{0\}) \times (\{0\} \times [0, +\infty)),$$

   the union of the unit circle, $x$-axis, and the positive part of the $y$-axis.

   (a) Describe the fundamental group of $X$ in terms of generators and relations.

   (b) Classify all 2-fold covering maps over $X$ (not necessarily connected).