Tamagawa numbers and special values of L–functions

Conference, May 20–31 2002, Institut Galilée, Université Paris 13

The aim of the conference is to explain the formulation of the Tamagawa number conjecture by Bloch and Kato, and the proof of this conjecture for Dirichlet motives. The background necessary for the formulation of the conjecture is explained by the main speakers, in eight series of talks (A–H). Series Z consists of talks by the participants (model: Arbeitsgemeinschaft Oberwolfach – thus, the subjects of these talks have been fixed in advance; see the program below). These talks will provide certain details of the general theory entering the proof for Dirichlet motives. Preferably, the speakers will *not* be experts in the field. Graduate students and post-docs are particularly encouraged to participate, and to apply for one of the talks of the series Z. In order to do so, write an e-mail to `tamagawa@math.univ-paris13.fr`, and establish a list of 3 talks (order of preference), each of which you would be ready to prepare (please do so before February 28). Instead of writing an e-mail, you may use the registration form on the conference home page

`http://zeus.math.univ-paris13.fr/~tamagawa/`

where more information of technical nature (concerning accommodation in particular) can be found as well. **NB:** Accommodation will be organized only for the main speakers, and for the participants giving a talk of the series Z. Please note the registration deadlines: April 15 for ordinary participants; February 28 for those wishing to give a talk of series Z.

Program

Our aim is to present a coherent picture of the ingredients of the Tamagawa number conjecture, and to discuss the example of Dirichlet characters. In order to achieve this goal, all speakers should follow closely the instructions given below, and use the notations as indicated. All talks are designed to last 60 minutes (see the time schedule at the end of this program). Please avoid to take more time. If you have questions, don't hesitate to contact one of the organizers: `guido.kings@mathematik.uni-regensburg.de`, or `wildesh@math.univ-paris13.fr`

General notations:

$F/\mathbb{Q}$ abelian number field
$K/\mathbb{Q}$ general number field
$E/\mathbb{Q}$ field of coefficients, $E_p = \mathbb{Q}_p \otimes E$, $E_\infty = E \otimes \mathbb{R}$
$\mathcal{O} \subset E$ ring of integers, $\mathcal{O}_p = \mathbb{Z}_p \otimes \mathcal{O}$
$M(r)$ motive, Tate twist r
V Artin motive
$V_{DR}, V_B, V_\ell, V_p, V_\infty$ DeRham, Betti, ℓ–adic, p–adic, and Hodge realizations of a motive
T_B, T_p $\mathbb{Z}$ (resp. $\mathbb{Z}_p$) –lattice in V_B (resp. V_p).

$V(\chi)$ Dirichlet motive for character χ
$M^\vee, V^\vee$ dual motive
$V_p^\vee$ E_p–dual
$T_p^* = \mathrm{Hom}_{\mathcal{O}_p}(T_p, E_p/\mathcal{O}_p)$ Pontryagin dual
k base field
A field of coefficients for the mixed sheaf theories: $A \in \{\mathbb{Q}, \mathbb{R}\}$ in Hodge theory, $A/\mathbb{Q}_\ell$ finite or $A = \overline{\mathbb{Q}_\ell}$ in the ℓ–adic theory
$Fr_{\mathfrak{p}}$ geometric Frobenius at $\mathfrak{p}$

$A_1 - A_7$: Mixed sheaves and absolute cohomology (K. Bannai, T. Saito, A. Shiho)

Purpose: For us, (locally constant) mixed sheaves will occur in two incarnations: *admissible variations of A–Hodge structure*, and *lisse ℓ-adic sheaves.* The three principal aims of this series are:

- the *explicit description* of (elements of) $\mathrm{Ext}^1(A(0), M)$, for a locally constant mixed sheaf M of Tate type over a smooth basis,
- the sheaf theoretical definition of the *polylogarithm* on $\mathbb{P}^1 - \{0, 1, \infty\}$, using the *Classification Theorem of Hain-Zucker* as its main conceptual ingredient, but avoiding the use of the general formalism of Grothendieck's functors on the derived category of not necessarily locally constant mixed sheaves,
- the *geometrical realization* of the polylogarithm, i.e., its identification with a class in absolute cohomology with coefficients.

Details:
$A_1 - A_2$**:** define *ℓ-adic Galois modules* and *mixed (graded-polarizable) Hodge structures.* Recall the relation between Yoneda-Ext^1 of Galois modules and the first Galois cohomology. The situation in Hodge theory needs more time: sketch Beilinson's definition of *absolute Hodge cohomology* ([Be2]), calculate $H^1_{\mathrm{abs}}(A(0), M)$, and explain the isomorphism between Yoneda-Ext^1 of Hodge structures and H^1_{abs} (give the explicit interpretation of this isomorphism in terms of the *period matrix*). Introduce the variants "over $\mathbb{R}$" of absolute Hodge cohomology.
Define lisse ℓ–adic sheaves and admissible variations of Hodge structures over a smooth basis X, with very little emphasis on admissibility; illustrate the point by discussing

$$\mathrm{Ext}^1_X(A(0), A(1))$$

in the admissible and the non–admissible category ([Wi1], IV, Thm. 3.7). Finally, explain how to describe elements of $\mathrm{Ext}^1_X(A(0), M)$: in the lisse ℓ–adic setting, the language of *torsors* will be most appropriate (for an example, see [Wi1], IV, Chapter 4).
Again, the situation in Hodge theory needs more time: assume that M is of Tate type, and show that elements of $\mathrm{Ext}^1_X(A(0), M)$ are determined by their period matrix (see [HuWi2], IV.3 and [Wi1], IV, Chapter 3).

$A_2 - A_6$**:** start by explaining the mixed structure on the completion of the group ring of the geometrical fundamental group $\pi_1(\overline{X}, \overline{x})$. Here, X is smooth over the base field k, and $x \in X(k)$. In the ℓ-adic setting, this is a formal consequence of the existence of the split exact sequence

$$1 \longrightarrow \pi_1(\overline{X}, \overline{x}) \longrightarrow \pi_1(X, x) \longrightarrow G_k \longrightarrow 1.$$

In the Hodge theoretic setting, this is Chen's theory of iterated integrals ([BrZ], 6.23). Define unipotent mixed sheaves. State and explain the Classification Theorem of Hain-Zucker ([BrZ], 7.19), as well as its analogue in the ℓ–adic setting. Define the *generic* pro-unipotent mixed sheaf Gen_x ([Wi1], p. 43), and give its universal property ([HuWi1], Thm. 2.1 d)).

Definition: The *logarithmic sheaf* on $\mathbb{G}_m$ is $\mathrm{Log} := \mathrm{Gen}_1$.

Identify $\mathrm{Gr}_\bullet^W \mathrm{Log}$. Up to here, the material is essentially contained in [Wi1], I, Chapters 2 and 3. Setting $\mathbb{U} := \mathbb{P}^1 - \{0, 1, \infty\}$, the rest of $A_2 - A_6$ will be devoted to the sheaf theoretical definition of the (small) polylogarithm:

Proposition-Definition:

(a) There is a canonical isomorphism

$$\mathrm{Ext}^1_{\mathbb{U}}(A(1), \mathrm{Log}(1)_{|\mathbb{U}}) \xrightarrow{\sim} A.$$

The pre-image of $1 \in A$ is defined as pol.

(b) Consider the isomorphism

$$\varphi : A(1) \xrightarrow{\sim} \mathrm{Gr}^W_{-2}\mathrm{Log}$$

given by the residue at zero, and the maps

$$\begin{array}{ccc} \mathrm{Ext}^1_{\mathbb{U}}(A(1), \mathrm{Log}(1)_{|\mathbb{U}}) & & \\ & \searrow (\pi_{\geq -2})_* & \\ & & \mathrm{Ext}^1(A(1), (W_{\geq -2}\ \mathrm{Log})(1)_{|\mathbb{U}}) \\ & \nearrow \varphi_* & \\ \mathrm{Ext}^1_{\mathbb{U}}(A(1), A(2)) & & \end{array}$$

Then $\mathrm{pol}^{(1)} := (\pi_{\geq -2}) * (\mathrm{pol}) = \varphi_*([1 - \mathrm{id}])$, where $[1 - \mathrm{id}]$ denotes the class of the invertible function 1-id on $\mathbb{U}$ in

$$\mathcal{O}^*(\mathbb{U}) \otimes \mathbb{A} \xrightarrow{\sim} \mathrm{Ext}^1_{\mathbb{U}}(A(0), A(1)) = \mathrm{Ext}^1_{\mathbb{U}}(A(1), A(2)).$$

(c) pol is uniquely determined by $\mathrm{pol}^{(1)}$, and even by the extension of ℓ–adic sheaves on $\mathbb{U} \otimes_{\mathbb{Q}} \overline{\mathbb{Q}}$ / local systems on $\mathbb{U}(\mathbb{C})$ underlying $\mathrm{pol}^{(1)}$.

For the proof, avoid the use of the general formalism of Grothendieck's functors (as in [Wi1], III, Thm.1.3). The central ingredient in the present approach is the calculation of the higher direct images of $\mathrm{Log}(1)_{|\mathbb{U}}$ under the projection

$$p : \mathbb{U} \longrightarrow \mathrm{Spec}\ k$$

([HuWi1], Thm.2.3): view p_* as a functor from unipotent locally constant mixed sheaves on $\mathbb{U}$ to mixed sheaves on Spec k. The Classification Theorem for $\mathbb{U}$ allows to compute the derived functor of p_*. For the details, see [Wi2].

$A_6 - A_7$**:** Sketch the content of [HuWi1], Sections 4 and 6, e.g., as in [HuWi2] VII.1. The formalism of Grothendieck's functors can be introduced axiomatically (see [Wi2]).

Notations: Follow the notations of [HuWi1], except for the coefficient field (use the letter A instead of F), and for the base ring (use the letter k instead of A).

Connections: The talks $E_1 - E_2$ depend on $A_1 - A_2$; $Z_6 - Z_7$ depend on $A_1 - A_6$; $D_3 - D_4$ depend on $A_6 - A_7$.

References: [Be2], [BrZ], [HuWi1], [HuWi2], [Wi1], [Wi2].

$B_1 - B_2$: Galois cohomology (O. Venjakob)

Purpose: Introduction of local and global *duality* in form of the *Poitou-Tate sequence*. Computation of local and global *Euler characteristics*.

Details: We need in B_1:

- local duality,
- the groups $H^i_f(K_v, V_p(r)), H^i_{/f}(K_v, V_p(r))$ as defined in [Fo1] 3.2, and their properties under local duality (proof in Z_4),
- local Euler characteristic.

B_2**:**

- Poitou-Tate sequence with $\mathcal{O}_p := \mathcal{O}\otimes\mathbb{Z}_p$ and $E_p := E\otimes\mathbb{Q}_p-$coefficients (see [FoPR] II.1.2.3 for the case of E_p-coefficients, but use notations $V_p^\vee, T_p^*$ as in "General notations"). The cohomology groups should be written $H^i(\mathcal{O}_k[1/pS], T_p(r))$ etc. with $K/\mathbb{Q}$ number field, S set of places of K.
- cohomology with compact support and its Euler characteristic ([Fo1] 4.2).

Notations: $H^i_f, H^i_{/f}$ in the local case, $H^i(\mathcal{O}_K[1/pS], -), H^i_c(\mathcal{O}_K[1/pS], -)$.

Connections: independent of former talks, B_1-B_2 is needed in $E_1-E_2, F_1-F_3, G_1-G_2, H_1-H_4, Z_4, Z_5, Z_8 - Z_{12}$.

References: [Fo1], [FoPR].

$C_1 - C_4$: p–adic Hodge theory and explicit reciprocity (D. Benois, C. Breuil)

Purpose: These talks review on the one hand the results from *p-adic Hodge theory* and on the other hand introduce the exponential map together with its explicit description in certain cases *(explicit reciprocity)*.

Details: $C_1 - C_4$: The rings B_{dR} and B_{cris}, together with the notions of cristalline and de Rham representations, will have been introduced in Z_3.
We need

- the isomorphism $D_{dR,\mathfrak{p}}(M_p) \cong (E \otimes_{\mathbb{Q}} K_{\mathfrak{p}}) \otimes M_{DR}$ and its compatibility with filtrations. ([Fo1] 6.2., [FoPR] I.2.2.). Here the normalization of φ should be as in [Fo1] 3.2.
- discussion of conjecture II.3.4.3 in [FoPR] and the cases where it is known.
- the exact sequence ([BlKa] prop. 1.7)

$$0 \to \mathbb{Q}_p \to B_{\text{cris},\mathfrak{p}} \to B_{\text{cris},\mathfrak{p}} \oplus B_{dR,\mathfrak{p}}/\text{Fil}^0 B_{dR,\mathfrak{p}} \to 0$$

- the maps exp and exp* and their properties ([BlKa] and [Ka2]); these must be given in C_1, as they are needed in Z_4.
- the definition of $H^1_f, H^1_{/f}$ (must be given in C_1 as it is needed in Z_4),
- the explicit reciprocity law as in [Ka2] 2.1.7 (see also [PR1], [ChCo]). Note that the argument in [Ka2] works also for a norm compatible system in $\mathcal{O}_{K_n}\backslash\{0\}$, K_n as in loc. cit.
- the explicit computation of the image of the cyclotomic element under exp* (see [Ka1] thm. 5.12).

Notations: use the notations from "General notations".

Connections: Input from Z_3. The results are used in $Z_4, Z_{11}, Z_{12}, F_1 - F_3, H_1 - H_4$.

References: [BlKa], [ChCo], [Ka2], [Fo1], [FoPR].

$D_1 - D_4$: Motives and motivic cohomology (M. Levine, F. Morel)

Purpose: These talks provide

- a review of Voevodsky's view of *motives* over a base field k,
- a sketch of the proof of the comparison results on *cyclotomic elements* in motivic and absolute cohomology.

Details:
$D_1 - D_2$: One may assume that k is of characteristic zero. For the definitions and results that need to be mentioned, see [Hu], Section 1.1.
We need:

- the definition of the *motivic cohomology with coefficients* as morphisms in the *triangulated category of (effective) geometrical motives*,
- the comparison result with Chow theory ([Vo], 4.2.6),
- as a consequence, the identification of the category of Chow motives in Grothendieck's sense as a full subcategory of our triangulated category,
- using Borel's theorem, the identification of the ranks of the

  $$H^i_{\mathcal{M}}(\mathrm{Spec}\ \mathbb{Q}, h^0(K)(j)),$$

  where $(i,j) \neq (1,1)$, and $h^0(K)$ is the motive of a number field K. State that

  $$H^1_{\mathcal{M}}(\mathrm{Spec}\ \mathbb{Q}, h^0(K)(1)) \cong K^*.$$

  In addition to what is said in [Hu], Section 1.1, we need the notion of an *Artin motive over k with coefficients in a field of coefficients E.* State that the category of such motives is equivalent to that of continuous representations of the Galois group on E-vector spaces of finite rank.

Example: *Dirichlet motives. A* Dirichlet character

$$\chi : (\mathbb{Z}/N\mathbb{Z})^* \to \mathbb{C}^*$$

defines a rank one Artin motive $V(\chi)$ with coefficients in the field E generated by the values of χ. In the category of these motives, $h^0(\mathbb{Q}_{(\mathcal{M}_N)})$is the direct sum of Dirichlet motives.

$D_3 - D_4$**:** State the comparison results on cyclotomic elements ([Be1], 7.1.5, [HuWi1], 9.7). The proof uses the construction of the *motivic polylogarithm*, and the *splitting principle* over roots of unity. The speaker *can* sketch the content of [HuWi1], Sections 8 and 9 (building on the material of the preceding sections of loc. cit., which have been treated in series A), but he is free to choose any alternative approach proving the result.

Notations: $H^i_{\mathcal{M}}(X, M)$ for the motivic cohomology of X with coefficients in M, "General notations" for $D_1 - D_2$. In $D_3 - D_4$, formulate the comparison results as in [HuKi], Thm. 5.3.1 and 5.3.2.

Connections: For $D_3 - D_4$, input from $A_1 - A_7, E_1 - E_2, Z_1 - Z_2$, and $Z_6 - Z_7$. $D_1 - D_2$ is needed in $E_1 - E_2, D_3 - D_4$ is needed in $H_3 - H_4$.

References: [Be1], [Hu], [HuKi], [HuWi1], [Vo].

$E_1 - E_2$: Regulators (J. I. Burgos)

Purpose: In these talks, two regulator maps on motivic cohomology will be defined: the *ℓ–adic regulator r_ℓ* to ℓ–adic cohomology (with coefficients $A = \mathbb{Q}_\ell$), and the *Beilinson regulator* r_∞ to absolute Hodge cohomology (with coefficients $A = \mathbb{Q}$ or $A = \mathbb{R}$). Certain refinements of r_ℓ in the case of cohomology of an Artin motive over $\mathbb{Q}$ will be discussed.

Details: In fact, r_ℓ and r_∞ are the maps on the level of morphism groups induced by ℓ–adic and Hodge theoretic *realization functors* r_ℓ and r_∞ on the triangulated category of motives, which was introduced in $D_1 - D_2$. Mention also the singular (or Betti) and de Rham realizations r_B and r_{DR}. Explain the abstract principles from [Hu], Section 2.1, necessary for the proof of [Hu], Cor. 2.3.4, Cor. 2.3.5. Observe that the purpose of loc. cit. is to prove the existence of realization functors to Huber's "derived" category of mixed realizations. This specializes to the ℓ–adic and Hodge theoretic realizations. However, we only need the existence of the latter; at least the construction of the ℓ–adic realization functor can be done in a more direct way than [Hu], Thm. 2.1.6; see loc. cit, Example on pp. 772/773.
At the end of the talks, specialize to the case of (twists of) Artin motives $V(r)$ over $\mathbb{Q}$ with coefficients in a number field E. Introduce the notation $H^i_{\mathcal{M}}(\mathrm{Spec}\,\mathbb{Z}, V(r))$: for $(i,r) \neq (1,1)$, this is identical to $H^i_{\mathcal{M}}(\mathrm{Spec}\,\mathbb{Q}, V(r))$, and $H^1_{\mathcal{M}}(\mathrm{Spec}\,\mathbb{Z}, V(1))$ is the direct factor of

$$\mathcal{O}_K^* \otimes_{\mathbb{Z}} E = E[\mathrm{Aut}\,K] \otimes_{E[\mathrm{Aut}\,K]} (\mathcal{O}_K^* \otimes_{\mathbb{Z}} E)$$

corresponding to the character of V, if V is a direct summand of $h^0(K)$. State Borel's theorem:

$$r_\infty \otimes \mathbb{R} : H^1_{\mathcal{M}}(\mathrm{Spec}\,\mathbb{Z}, V(r)) \otimes \mathbb{R} \longrightarrow H^1_{\mathrm{abs}}(\mathrm{Spec}\,\mathbb{R}, V_\infty(r))$$

is an isomorphism for $r > 1$. For $r = 0$, we have the Betti regulator

$$r_B : H^0_{\mathcal{M}}(\mathrm{Spec}\,\mathbb{Z}, V) \longrightarrow (V_B \otimes \mathbb{R})^+.$$

Recall from $A_1 - A_2$ that the latter group can be identified with $H^1_{\mathrm{abs}}(\mathrm{Spec}\,\mathbb{R}, V_\infty(1))$. For all primes ℓ, the ℓ–adic regulator factorizes through

$$r_\ell : H^1_{\mathcal{M}}(\mathrm{Spec}\,\mathbb{Z}, V(r)) \longrightarrow H^1(\mathrm{Spec}\,\mathbb{Z}[1/S\ell], V_\ell(r)),$$

for a finite set S of primes. Quote [So], Thm. 1: $r_\ell \otimes \mathbb{Q}_\ell$ is an isomorphism for $r > 1$. Define $H^1_f(\mathrm{Spec}\,\mathbb{Z}[1/S\ell], V_\ell(r))$ as the group of those classes in $H^1(\mathrm{Spec}\,\mathbb{Z}[1/S\ell], V_\ell(r))$ which map to $H^1_f(\mathbb{Q}_v, V_\ell(r))$ for *all* v (including $v = \ell$). r_ℓ factors through

$$r_\ell : H^1_{\mathcal{M}}(\mathrm{Spec}\,\mathbb{Z}, V(r)) \longrightarrow H^1_f(\mathrm{Spec}\,\mathbb{Z}[1/S\ell], V_\ell(r)),$$

and $r_\ell \otimes \mathbb{Q}_\ell$ is an isomorphism for all r. (This follows from the above case $r > 1$, compatibility of H^1_f with local duality, and the explicit shape of the exponential for $r = 1$ (talks B_1 and Z_4).)

Notations: use the notations from "General notations", and those introduced in $A_1 - A_2$ and $D_1 - D_2$. Introduce $H^i_{\mathcal{M}}(\mathrm{Spec}\,\mathbb{Z}, V(r))$ and $H^1_f(\mathrm{Spec}\,\mathbb{Z}[1/S\ell], V_\ell(r))$.

Connections: Input from $A_1 - A_2, B_1, D_1 - D_2, Z_4$. The talks $F_1 - F_3, D_3 - D_4$ and $H_1 - H_4$ depend on $E_1 - E_2$.

References: [Hu], [So].

$F_1 - F_3$: Formulation of the conjectures (A. Huber)

Purpose: These talks will formulate the *Tamagawa number conjecture* of Bloch and Kato in its *absolute* and *equivariant* form (Kato, Burns–Flach). We want to concentrate on the case of Artin motives and mention the general case only in passing.

Details: We need

- a short review of "det",
- the definition of $H^i(\mathbb{Z}[1/p], T_p) := H^i(\mathbb{Z}[1/pS], j_* T_p)$ and an explanation why this is independent of S (in the case of Dirichlet motives),
- the definition of $R\Gamma_f, R\Gamma_c$ (the necessary background is explained in $B_1 - B_2, C_1 - C_3$ and Z_4),
- a discussion of the identification of det $R\Gamma_f(\mathbb{Q}_v, V_p(r))$,
- the computation of the cohomology groups of $R\Gamma_f(\mathbb{Z}[1/p], V_p(r))$,
- the formulation of the absolute and equivariant conjecture (cf. [Fo1], [BuFl]),
- the formulation of the theorem of Burns-Greither / Huber-Kings ([BuGr], [HuKi]),
- the comparison of det $R\Gamma_c$ with det $R\Gamma$,
- compatibility under the equivariant functional equation,
- the relation to the Lichtenbaum conjecture.

Notations: $\Delta_f(K/\mathbb{Q}, V(r))$ equivariant fundamental line, rest as in "General notations".

Connections: Input from $B_1 - B_2, C_1 - C_3$ and Z_4. The talks $H_1 - H_4, Z_9, Z_{11}, Z_{12}$ depend on $F_1 - F_3$.

References: [BuGr], [BuFl], [Fo1], [HuKi].

$G_1 - G_2$: Iwasawa theory and Euler systems (S. Howson)

Purpose: Here we discuss Λ–modules in general and introduce the Λ–modules $\mathbb{H}^1(T_p)$ and $\mathbb{H}^i(T_p^*)$. Moreover, *Euler systems* are defined and the main theorems about them explained.

Details:

G_1: We need:

- the definition of $\Lambda = \varprojlim_n \mathcal{O}_p[\mathrm{Gal}\ (\mathbb{Q}_n/\mathbb{Q})]$, where $\mathcal{O}_p$ is as in "General notation" and $\mathbb{Q}_n/\mathbb{Q}$ runs through the layers of the cyclotomic $\mathbb{Z}_p$-extension,

- the isomorphism $\Lambda \cong \mathcal{O}_p[[t]]$,
- the classification of Λ–modules (up to pseudo-isomorphism),
- the relation between $\det_\Lambda$ (as introduced in F_1) and char_Λ (characteristic polynomial). This is explained in [Ka1] prop. 6.1.
- introduction of the Λ–modules $\mathbb{H}^i(T_p)$ (e.g. [HuKi] 4.1.2, [PR2] 1.3, the notation should be as in [HuKi]).

G_2:

- the definition of Euler systems ([Ru] 2.1.1; the case $K = \mathbb{Q}$, $K_\infty =$ the cyclotomic $\mathbb{Z}_p$–extension suffices). The Euler factor should be denoted by

  $$P_v(V_p, t) := \det_{Ep}(1 - Fr_v \cdot t \mid V_p^{I_v})$$

  where $v \nmid p, Fr_v$ is the geometric Frobenius and I_v the inertia group at v.
- as an example the elements $c_r(\zeta_m)$ and $c_r(\zeta_m)(\chi)$ from Z_2 (see also [HuKi] 3.1.2 and 3.1.3.) should be discussed.
- the consequences of non-torsion Euler systems should be discussed. This is theorem 2.2.3 and 2.3.3 in [Ru]. (Please follow the notations in [HuKi] proof of 3.3.1 and thm. 4.3.1. It would also be useful to explain these two applications).

Notations: $\Lambda, \mathbb{Q}_n, \mathbb{Q}_\infty, \mathbb{H}^i_{gl}(T_p(k)), \mathbb{H}^i_{gl}(T_p^*(1-k)), \mathbb{H}^i_{\mathrm{loc}}(T_p(k)), \mathbb{H}^i_{\mathrm{loc}}(T_p^*(1-k)), \mathbb{H}^2_{gl,0}(T_p(k))$.

Connections: Input from Z_2. The results are relevant in $H_1 - H_4, Z_8, Z_{10}$.

References: [HuKi], [Ka1], [PR2], [Ru].

$H_1 - H_4$: Proof of the Tamagawa number conjecture for Dirichlet motives (A. Schmidt, M. Spiess)

Purpose: This series of lectures is devoted to the case of Dirichlet motives. Its aim is to put all the prerequisite work together to give a proof of the Tamagawa number conjecture for Dirichlet motives.

Details: We need (consult also the overview [HuKi] 2.1):

- the formulation of the main conjecture in the equal/unequal parity case (see e.g. [HuKi] 4.2.2, 4.2.4) (input from $G_1 - G_2$),
- the divisibility statement, which follows from the Euler system (see e.g. [HuKi] 4.3.3) (the facts about Iwasawa modules needed in the proof are treated in Z_{10}),
- the proof of the main conjecture for characters χ with $\chi(-1) = (-1)^{k-1}$ (see e.g. [HuKi] 4.4.1),

- the proof of the Tamagawa number conjecture in the case $\chi(-1) = (-1)^r$ and $r \neq 0$ (for $r = 0$ one needs here $\chi(p) \neq 1$, which will be removed later). Input from D_3, D_4.
- from the functional equation (input from Z_{11}, Z_{12}), one deduces the case $\chi(-1) = (-1)^{r-1}$ for $r \neq 0, 1$. (Note that the equivariant Bloch-Kato conjecture is only necessary to treat the case $r = 0, 1$.)
- one treats the case $r = 0$, $\chi(-1) = -1$ and $\chi(p) \neq 1$,
- one treats the case $r = 1$, $\chi(-1) = 1$,
- the missing case $r = 0$, $\chi(p) = 1$ follows then again from the functional equation.
- the equivariant case ([BuGr]).

Notations: The notations should follow the "General notations", and the notations from the inputs (often identical to the ones in [HuKi]).

Connections: Input from all talks. Note that talks $D_3 - D_4$ and $Z_{11} - Z_{12}$ take place *after* talks $H_1 - H - 2$.

References: [BuGr], [HuKi].

Z_1: Higher logarithms and Dirichlet L-functions

Purpose: This talk establishes the basic properties of and the relation between higher logarithms $Li_s(z)$ and Dirichlet L–functions $L(\chi, s)$.

Details: Introduce the *higher logarithms*

$$Li_s(z) := \sum_{n=1}^{\infty} \frac{z^n}{n^s},$$

for $Re(s) \geq 1$ and $|z| \leq 1 (|z| < 1$ if $Re(s) = 1)$. For an integer $s = k \geq 1$, establish the main analytic properties of the Li_k: representation by (iterated) integrals and (multivalued) continuation to $\mathbb{P}^1 - \{0, 1, \infty\}$ ([BeD], page 97), monodromy around 0 and 1 ([Wi1], IV, Prop. 3.4 (a), (b)).
Introduce the notions of *Dirichlet character*, of its *conductor*, of a *primitive* Dirichlet character, and of the *L-function* $L(\chi, s)$ of a (primitive) Dirichlet character. Using the fact that the *Hurwitz zeta function* admits a meromorphic continuation to the complex plane, which is analytic except for a simple pole at $s = 1$ with residue 1 ([Ap], Thm. 12.4), deduce the analytic properties of $L(\chi, s)$ ([Ap], Thm. 12.5). Briefly discuss *special values* $L(\chi, 1 - k)$ ([Wa], Thm. 4.2).
Give the statement, and (if possible) sketch the solution of Exercice 8 (b) of [Ko], page 78. Using the fact that Γ has a simple pole of residue $(-1)^{k-1} \frac{1}{(k-1)!}$ at $s = 1 - k$, deduce the following two results:

Theorem: Let $k \geq 1$ be an integer, and $\chi : (\mathbb{Z}/N\mathbb{Z})^* \to \mathbb{C}^*$ a primitive Dirichlet character satisfying $\chi(-1) = (-1)^{k-1}$. (Hence $L(\chi, 1-k) = 0$.) Then

$$\frac{N^{k-1}(k-1)!}{2} \sum_{m=1}^{N} \chi(m) Li_k(e^{\frac{2\pi i m}{N}}) = (2\pi i)^{k-1} L'(\chi, 1-k).$$

Theorem:

(a) For $|z| = 1$, the function $s \mapsto \mathrm{Li}_s(z)$ $(\mathrm{Re}(s) > 1)$ admits a meromorphic continuation to $\mathbb{C}$.

(b) Let $r \geq 1$ be an integer, and $\chi : (\mathbb{Z}/N\mathbb{Z})^* \to \mathbb{C}^*$ a primitive Dirichlet character satisfying $\chi(-1) = (-1)^r$. Then

$$\sum_{m=1}^{N} \chi(m) \mathrm{Li}_{1-r}(e^{\frac{2\pi i m}{N}}) = 2(-\frac{N}{2\pi i})^r (r-1)! L(\chi, r).$$

Notations: Li_s for the higher logarithms, $L(\chi, s)$ for the L-function of χ.

Connections: Talks Z_6 and $D_3 - D_4$ depend on this talk.

References: [Ap], [BeD], [Ko], [Wa], [Wi1].

Z_2: Cyclotomic elements in Galois cohomology

Purpose: This talk establishes the basic properties of the Soulé–Deligne cyclotomic elements. In particular, *Norm compatibility* will be discussed.

Details: Start with the polynominal identity

$$\prod_{\alpha^k=1} (1 - \alpha X) = 1 - X^k,$$

a central tool in a number of proofs. Fix a collection $(\zeta_m)_{m \geq 1}$ of primitive m–th roots of unity satisfying $\zeta_{km}^k = \zeta_m$. For any prime ℓ and any m prime to ℓ, we denote by Fr_ℓ the *geometric* Frobenius at ℓ in $\mathrm{Gal}(\mathbb{Q}(\zeta_m)/\mathbb{Q})$, i.e., the inverse of the automorphism determined by $\zeta_m \mapsto \zeta_m^\ell$. Using standard facts about cyclotomic fields (e.g. the first two chapters of [Wa]), prove the following:

Proposition: For every prime ℓ, we have

$$N_{\mathbb{Q}(\zeta_{\ell m})/\mathbb{Q}(\zeta_m)}(1 - \zeta_{\ell m}) = \begin{cases} 1 - \zeta_m & \text{if } \ell \mid m \\ (1 - \zeta_m)^{(1 - Fr_\ell)} & \text{if } \ell \nmid m \text{ and } m > 1 \\ \ell & \text{if } m = 1 \end{cases}$$

Fix a prime p. Recall that $1 - \zeta_{p^n m}$ is a unit in $\mathbb{Z}[\zeta_{p^n m}][1/p]$ for every $n, m \geq 1$. We shall consider units in this ring as elements of the first etale cohomology group

$$H^1(\mathbb{Z}[\zeta_{p^n m}][1/p], \mathbb{Z}/p^n\mathbb{Z}(1)).$$

More generally, consider $(1-\zeta_{p^n m})\otimes(\zeta_{p^n})^{\otimes r-1}$ as an element of

$$H^1(\mathbb{Z}[\zeta_{p^n m}][1/p], \mathbb{Z}/p^n\mathbb{Z}(r)),$$

for $r \in \mathbb{Z}$.

Notation: The image of this element under corestriction (= the norm) from $\mathbb{Z}[\zeta_{p^n m}][1/p]$ to $\mathbb{Z}[\zeta_m][1/p]$ is denoted by $c_r(\zeta_m)_n$:

$$c_r(\zeta_m)_n \in H^1(\mathbb{Z}[\zeta_m][1/p], \mathbb{Z}/p^n\mathbb{Z}(r)),$$

$m, n \geq 1, r \in \mathbb{Z}$.
Using the product formula cores $(x \otimes \text{res }(y)) = \text{cores }(x) \otimes y$, prove that the $c_r(\zeta_m)_n$ are compatible with respect to the maps $\mathbb{Z}/p^n\mathbb{Z} \to \mathbb{Z}/p^{n-1}\mathbb{Z}$.

Definition: (see [So], page 381 - 383.)
The *Soulé–Deligne cyclotomic elements*

$$c_r(\zeta_m) \in \varprojlim_n H^1(\mathbb{Z}[\zeta_m][1/p], \mathbb{Z}/p^n\mathbb{Z}(r))$$

are defined to be

$$c_r(\zeta_m) := \varprojlim_n c_r(\zeta_m)_n,$$

$m \geq 1, r \in \mathbb{Z}$.

State and prove the following:
Proposition: (Norm compatibility.)
For every prime ℓ, corestriction from $\mathbb{Q}(\zeta_{\ell m})$ to $\mathbb{Q}(\zeta_m)$ maps $c_r(\zeta_{\ell m})$ to

$$\begin{cases} c_r(\zeta_m) & \text{if} \quad \ell \mid pm \\ (1-\ell^{r-1}\mathrm{Fr}_\ell)c_r(\zeta_m) & \text{if} \quad \ell \nmid pm \end{cases}$$

We need also the following variant:

Notation: (see [Ka1], §5.)

$$\tilde{c}_r(\zeta_m) := \begin{cases} c_r(\zeta_m) & \text{if} \quad p \mid m \\ \sum_{i\geq 0}(p^{r-1})^i c_r(\zeta_m^{p^{-i}}) & \text{if} \quad p \nmid m, r \geq 2 \\ 1-\zeta_m & \text{if} \quad r = 1 \\ -\sum_{i\geq 0}(p^{1-r})^i c_r(\zeta_m^{p^i}) & \text{if} \quad p \nmid m, r \leq 0 \end{cases}$$

In the second case, $\zeta_m^{p^{-i}}$ denotes the unique m–th root of unity whose p^i–th power equals ζ_m.
Note that for $p \nmid m$, we have $(1-p^{r-1}\mathrm{Fr}_p)\tilde{c}_r(\zeta_m) = c_r(\zeta_m)$.

State and prove the following:

Proposition:

(a) Let $m, n \geq 1, r \geq 2$. Then the following identity holds in $H^1(\mathbb{Z}[\zeta_{p^n m}][1/p], \mathbb{Z}/p^n\mathbb{Z}(r))$:

$$\sum_{\beta^{p^n}=\zeta_m} (1-\beta) \otimes (\beta^m)^{\otimes r-1} = \begin{cases} \sum_{i=0}^{n} (p^{r-1})^i c_r(\zeta_m^{p^{-i}})_{n-i} & \text{if} \quad p \nmid m \\ c_r(\zeta_m)_n & \text{if} \quad p \mid m \end{cases}$$

(b) Let $m \geq 1, r \geq 1$. Then

$$\varprojlim_n \left(\sum_{\beta^{p^n}=\zeta_m} (1-\beta) \otimes (\beta^m)^{\otimes r-1} \right) = \tilde{c}_r(\zeta_m)$$

$$\text{in } \varprojlim_n H^1(\mathbb{Z}[\zeta_{p^\infty m}][1/p], \mathbb{Z}/p^n\mathbb{Z}(r)).$$

For the proof, see [HuKi], Lemma 3.1.6.

Notations: Fr_ℓ for the *geometric* Frobenius, $c_r(\zeta_m)$ for the Soulé-Deligne cyclotomic elements, $\tilde{c}_r(\zeta_m)$ for their variants after Kato.

Connections: Talks Z_7, G_2 and $D_3 - D_4$ depend on this talk.

References: [HuKi], [Ka1], [So], [Wa].

Z_3: De Rham and cristalline Galois representations

Purpose: In this talk the rings B_{DR}, B_{cris} and some of their properties are introduced. Moreover, one defines the functors D_{DR} and D_{cris}, and the notions of *de Rham* and *cristalline representations*. The example $V_p(\chi)$ will be discussed.

Details: We need

- a short review of the construction of $B_{DR,\mathfrak{p}}$ ([Fo2] 2.3, 2.4, 2.8, 2.17) ($K_\mathfrak{p}$ local field of char 0, finite extension of $\mathbb{Q}_p$),
- the properties of B_{DR} ([Fo3] 1.5.4., 1.5.5), $G_\mathfrak{p} := \mathrm{Gal}(\overline{K}_\mathfrak{p}/K_\mathfrak{p})$–action, $B_{DR,\mathfrak{p}}^{G_\mathfrak{p}} = K_\mathfrak{p}$,
- a short review of the construction of $B_{\text{cris},\mathfrak{p}}$ ([Fo3] 2.3.3., 2.3.4), $G_\mathfrak{p}$–action, $B_{\text{cris},\mathfrak{p}}^{G_\mathfrak{p}} = K_{\mathfrak{p},0} = \mathrm{Quot}\, W(k_\mathfrak{p})$ where $k_\mathfrak{p}$ in the residue class field of $K_\mathfrak{p}$,
- definition of D_{DR}, D_{cris} ([Fo4] 3.7, 5.1.4),
- discuss the example $T_p(\chi)$. When is it de Rham, when cristalline?
- remark that $D_{\text{cris},\mathfrak{p}}(h^0(K_\mathfrak{p})) = K_{\mathfrak{p},0}$.

Notations: $K_{\mathfrak{p}}$ local field, finite extension of $\mathbb{Q}_p$, $k_{\mathfrak{p}}$ residue class field of $K_{\mathfrak{p}}, K_{\mathfrak{p},0} = \text{Quot } W(k_{\mathfrak{p}}), B_{\text{cris},\mathfrak{p}}, B_{DR,\mathfrak{p}}, D_{\text{cris}}, D_{DR}$.

Connections: This talk is needed in the series $C_1 - C_4$. Please coordinate your talk with the speakers of $C_1 - C_4$.

References: [Fo2], [Fo3], [Fo4].

Z_4: The exponential map for connected p-divisible groups and local duality

Purpose: This talk explains the *exponential map* in the case of (connected) p-divisible groups and proves the duality results for H^1_f and $H^1_{/f}$.

Details: The talk should cover the example 3.10.1 in [BlKa] (if necessary consult [Sch2] §2) and the proof of proposition 3.8 in [BlKa].

Notations: follow "General notations" for the Pontryagin dual and the $\mathbb{Q}_p$–dual.

Connections: Input from $C_1 - C_4$, results are needed in Z_5, Z_9.

References: [BlKa], [Sch2].

Z_5: Cohomological interpretation of the class number formula

Purpose: This talk establishes an exact sequence, which allows to interpret the group of units $\mathcal{O}^*_F$ and the class group in terms of Galois cohomology.

Details: First one needs the identification $H^1_f(K, \mathbb{Z}_p(1)) = \mathcal{O}^*_K \otimes \mathbb{Z}_p$, for a local field $K/\mathbb{Q}_p$ (explained in Z_4, see also [HuKi] A.0.4). Then explain that the map $H^1(K, \mathbb{Z}_p(1)) \longrightarrow H^1_{/f}(K, \mathbb{Z}_p(1))$ can be identified with the valuation $v : K^* \otimes \mathbb{Z}_p \longrightarrow \mathbb{Z}_p$ ([HuKi] A.0.5). Finally prove [HuKi] A.0.6 and explain the input from [Sch1].

Notations: follow "General notations".

Connections: Input from Z_4. The results are needed in Z_9.

References: [HuKi], [Sch1].

Z_6: Hodge theoretic realization of the polylogarithm

Purpose: This talk shows that the higher logarithms can be used to describe a variation of Hodge structure on $\mathbb{P}^1 - \{0, 1, \infty\}$, and that this variation equals the polylogarithm pol introduced in series A.

Details: Start by showing how the analytic properties of the functions Li_k can be encoded in a (pro-)variation on $\mathbb{P}^1 - \{0, 1, \infty\}$([BeD], page 97–100, but use the normalization of [HuWi2], Section IV and [Wi1], IV, Chapter 3 (setting $N = 1$ in [Wi1])). The main result is Beilinson's theorem on the explicit shape of the specialization of pol at roots of unity ([HuWi1], Thm. 2.5). For this, one has to show that the variation described by the Li_k corresponds to the extension pol which was introduced in talks $A_5 - A_6$ ([Wi1], IV, Thm. 3.5).

Notations: see the notations of talks Z_1 and $A_1 - A_6$.

Connections: Input from talks Z_1 and $A_1 - A_6$. Talks $D_3 - D_4$ depend on this talk.

References: [BeD], [HuWi1], [HuWi2], [Wi1].

Z_7: ℓ–adic realization of the polylogarithm.

Purpose: This talk shows that the cyclotomic elements describe the specialization of the ℓ–adic polylogarithm pol at roots of unity.

Details: The main result is Beilinson's theorem on the explicit shape of the specialization of the ℓ–adic version of the polylogarithm at roots of unity ([HuWi1], Thm.2.6).
Use the language of torsors introduced in $A_1 - A_2$ to give the explicit description of the extension pol which was introduced in talks $A_5 - A_6$ ([Wi1], IV, Th. 4.2, setting $N = 1$). Deduce Beilinson's theorem as in loc. cit., taking into account the remark made before [HuWi1], Thm. 2.6.

Notations: see the notations of talks Z_2 and $A_1 - A_7$. In the formulation of Beilinson's theorem, use the elements $\tilde{c}_r(\zeta_m)$ introduced in talk Z_2.

Connections: Input from talks Z_2 and $A_1 - A_7$. Talks $D_3 - D_4$ depend on this talk.

References: [HuWi1], [Wi1].

Z_8: Some properties of Λ–modules

Purpose: The aim of this talk is to give criteria when the Λ–modules $\mathbb{H}^2(T_p)$ are torsion.

Details: Explain the proof of proposition 1.3.2 in [PR2] using the notations introduced in $G_1 - G_2$. Discuss the Leopoldt conjecture (loc. cit. p. 28) and explain why it is true for $T_p(\chi)$ ([HuKi] 4.1.4).

Notations: follow the notations from $G_1 - G_2$ and from "General notations".

Connections: The results are needed in $H_1 - H_4$.

References: [HuKi], [PR2].

Z_9: The Bloch-Kato conjecture in the class number formula case

Purpose: With the help of Z_5 the Bloch-Kato conjecture for the motive $h^0(F)$ and $r = 0, 1$ will be proved.

Details: Follow the exposition in [HuKi] 2.3.1. An additional input is loc. cit. prop. A.0.10, which should also be proved.

Notations: follow "General notations".

Connections: The talk relies on Z_5 and is needed in $H_1 - H_4$.

References: [HuKi].

Z_{10}: Computation of some Λ-modules

Purpose: For the proof of the main conjecture of Iwasawa theory one needs some precise information about certain Λ-modules. These are provided in this talk.

Details: The input in the proof of the main conjecture consists in proposition 6.1.2, 6.2.1 and 6.3.3 in [HuKi]. The proofs of these propositions covers the whole paragraph 6 in [HuKi], which should be followed.

Notations: follow the notations in [HuKi] and "General notations".

Connections: The definition of the relevant Iwasawa-modules is given in $G_1 - G_2$. The results are needed in $H_1 - H_4$.

References: [HuKi].

$Z_{11} - Z_{12}$: The functional equation

Purpose: These two talks prove the compatibility of the Bloch-Kato conjecture under the functional equation. This allows to translate the results from the case $r \leq 0$ to $r \geq 1$ and vice versa.

Details: The talks should cover appendix B in [HuKi]. The formulation of the compatibility result will be given in $F_1 - F_3$ in its equivariant form. Note that the equivariant case is only needed to treat the cases $r = 0, 1$.

Notations: follow the notation in loc. cit.

Connections: The result is needed in $H_1 - H_4$.

References: [HuKi].

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Time schedule

Monday, May 20: Arrival of the speakers and participants

Tuesday, May 21:

10h00 - 11h00: Registration

11h00 - 12h00: A_1*: Mixed sheaves and absolute cohomology I* (K. Bannai, T. Saito, A. Shiho)

12h30 - 13h30: A_2*: Mixed sheaves and absolute cohomology II*

13h30 - 15h00: Lunch break

15h00 - 16h00: Z_1*: Higher logarithms and Dirichlet L-functions* (to be attributed)

16h30 - 17h30: Z_2*: Cyclotomic elements in Galois cohomology* (to be attributed)

Wednesday, May 22:

9h30 - 10h30: A_3*: Mixed sheaves and absolute cohomology III*

11h00 - 12h00: B_1*: Galois cohomology I* (O. Venjakob)

12h30 - 13h30: Z_3*: De Rham and cristalline Galois representations* (to be attributed)

13h30 - 15h00: Lunch break

15h00 - 16h00: C_1*: p-adic Hodge theory and explicit reciprocity I* (D. Benois, C. Breuil)

16h30 - 17h30: A_4*: Mixed sheaves and absolute cohomology IV*

Thursday, May 23:

9h30 - 10h30: D_1*: Motives and motivic cohomology I* (M. Levine, F. Morel)

11h00 - 12h00: B_2*: Galois cohomology II*

12h30 - 13h30: Z_4*: The exponential map for connected p-divisible groups and local duality* (to be attributed)

13h30 - 15h00: Lunch break

15h00 - 16h00: Z_5*: Cohomological interpretation of the class number formula* (to be attributed)

16h30 - 17h30: A_5*: Mixed sheaves and absolute cohomology V*

Friday, May 24:

9h30 - 10h30: D_2: *Motives and motivic cohomology II*

11h00 - 12h00: A_6: *Mixed sheaves and absolute cohomology VI*

12h30 - 13h30: Z_6: *Hodge theoretic realization of the polylogarithm* (to be attributed)

13h30 - 15h00: Lunch break

15h00 - 16h00: Z_7: *ℓ–adic realization of the polylogarithm* (to be attributed)

16h30 - 17h30: E_1: *Regulators I* (J.I. Burgos)

Monday, May 27:

9h30 - 10h30: E_2: *Regulators II*

11h00 - 12h00: A_7: *Mixed sheaves and absolute cohomology VII*

12h30 - 13h30: C_2: *p-adic Hodge theory and explicit reciprocity II*

13h30 - 15h00: Lunch break

15h00 - 16h00: C_3: *p-adic Hodge theory and explicit reciprocity III*

16h30 - 17h30: F_1: *Formulation of the conjectures I* (A. Huber)

Tuesday, May 28:

9h30 - 10h30: C_4: *p-adic Hodge theory and explicit reciprocity IV*

11h00 - 12h00: F_2: *Formulation of the conjectures II*

12h30 - 13h30: F_3: *Formulation of the conjectures III*

13h30 - 15h00: Lunch break

15h00 - 16h00: G_1: *Iwasawa theory and Euler systems I* (S. Howson)

16h30 - 17h30: Z_8: *Some properties of Λ–modules* (to be attributed)

Wednesday, May 29:

9h30 - 10h30: G_2: *Iwasawa theory and Euler systems II*

11h00 - 12h00: Z_9: *The Bloch-Kato conjecture in the class number formula case* (to be attributed)

12h30 - 13h30: H_1: *Proof of the Tamagawa number conjecture for Dirichlet motives I* (A. Schmidt, M. Spiess)

13h30: Lunch

Thursday, May 30:

9h30 - 10h30: Z_{10}: *Computation of some* Λ*-modules* (to be attributed)

11h00 - 12h00: H_2: *Proof of the Tamagawa number conjecture for Dirichlet motives II*

12h30 - 13h30: Z_{11}: *The functional equation I* (to be attributed)

13h30 - 15h00: Lunch break

15h00 - 16h00: Z_{12}: *The functional equation II* (to be attributed)

16h30 - 17h30: D_3: *Motives and motivic cohomology III*

Friday, May 31:

9h30 - 10h30: D_4: *Motives and motivic cohomology IV*

11h00 - 12h00: H_3: *Proof of the Tamagawa number conjecture for Dirichlet motives III*

12h30 - 13h30: H_4: *Proof of the Tamagawa number conjecture for Dirichlet motives IV*

13h30: Lunch

14h30: End of the conference