

ENCYCLOPEDIA OF MATHEMATICS AND ITS APPLICATIONS

An Algebraic Introduction to K -Theory

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Corrections

p.(i), and back cover, line 7: replace “classic groups” by “classical groups”

p. 36, in (1.35)(ii): replace A^t by A^τ

p. 37, line 5: replace R^2 by \mathbb{R}^2

p. 39, last line: insert the word “nontrivial”, so it becomes:

“Every nontrivial left noetherian ring has IBN.”

p. 77, line 19: replace $A = f^n(A) \oplus \ker(f^n)$ by
 $A \cong f^n(A) \oplus \ker(f^n)$

p. 96, line 5: replace $M = 1$ by $m = 1$

p. 100, last line: replace P'' by P'_0

p. 109, second display of (4.8): replace $ij - ji = -2ij$ by $ij - ji = 2ij$

p. 110, second line before example (ii): replace “resolved” by “affirmed”

p. 129, line 10: replace

“are intersections of (a nonempty set of) maximal ideals of R ” by

“are intersections of R with a set of maximal ideals of R ”

p. 130, line -7: replace $a'_0 = 0, a', \dots, a'_i$ by $a'_0 = 0, a'_1, \dots, a'_i$

p. 136, line -5: replace “ $\overset{d}{\cup}$ and X ” by “ $\overset{d}{\cup}$ and \times ”

p. 139, second display: replace $\phi^H = \phi$ by $\emptyset^H = \emptyset$

p. 139, last line: the last sum should be $\sum n_i[G/H_i]$

p. 143, line -5: replace “ $m \otimes (-) : M \rightarrow M \otimes_R N$ ” by
“ $m \otimes (-) : N \rightarrow M \otimes_R N$ ”

p. 147, line 6: replace $f : M \rightarrow M$ by $f : M \rightarrow M'$

p. 152, line 2: replace $(b' \perp b')$ by $(b \perp b')$

p. 152, second paragraph, line 5: replace $\text{cl}(M, f)$ by $c(M, b)$

- p. 152, second paragraph, line 6: replace $[M, f]$ by $[M, b]$
- p. 152, line -3: replace (3.11) by (3.12)
- p. 166, line 14: replace “Then” by “For $\epsilon_{j1} \in R^{m \times n}$ ”
- p. 177, Proposition (6.28); replace “some” by “an IBN”
- p. 188, line -9: replace “zero divisor of A ” by “zero divisor of A^{op} ”
- p. 194, line -2: replace A^n by A^r
- p. 194, line -1: replace $\frac{A^n}{u(A^m)}$ by $\frac{A^r}{u(A^m)}$
- p. 195, line 2: replace $\frac{A^n}{u(A^m)}$ by $\frac{A^r}{u(A^m)}$
- p. 228, line 7: replace “positive” by “arbitrary”
- p. 234, lines 2 and 3: remove commas before $P_n^{e_n}$ (three occurrences)
- p. 234, line 13: replace $T(B)$ by $\mathcal{T}(B)$
- p. 269, line -4 of the last paragraph of the proof of (8.13) should begin with $\overline{e_1}$ instead of \overline{e} , and the equation in the next line should be $e_2^2 = e^2 - ee_1 - e_1e + e_1^2$
- p. 273, line 10: replace X_i by x_i
- p. 279, first line of proof: replace $A^n = A \overset{\bullet}{\oplus} \cdots \overset{\bullet}{\oplus} A$ by $A^n = A \oplus \cdots \oplus A$
- p. 290, last line before (8.40) should begin $GL_{n_i}(F)$, not $GL_{n_i}(G)$
- p. 291, line 6: replace ϵ_{ij} by ϵ_{j1}
- p. 291, line 12: change the subscript from $1j$ to $j1$, so the line becomes
“= the first column of $(\pi_i \circ \theta)(g)\epsilon_{j1}$ ”
- p. 291, line -4: make the inequality strict, so it becomes
“ $\dim_E EG < \dim_F FG$ ”
- p. 292, line -9: replace the sentence
“Only one of these, $S_{\mu'}$, contains EGe ; but $EGe \subseteq EG\varepsilon$ because $EG\varepsilon$ contains E and FGe .” by
“Since $FGe \subseteq S_{\mu} = FG\varepsilon$, we also have $EGe \subseteq EG\varepsilon$; so the simple component $S_{\mu'}$ of EG lies in $EG\varepsilon$.”
- p. 293, The proof of (8.47) should be:
- Proof.* Since F is a splitting field, there is an F -algebra isomorphism θ as in (8.46), and by (8.41),

$$\theta = [\pi_1 \circ \theta \quad \cdots \quad \pi_r \circ \theta] = [\widehat{\mu}_1 \quad \cdots \quad \widehat{\mu}_r]$$

where μ_i is afforded by L_{i1} . If ν_1, \dots, ν_r are listed in the order for which ν_i is similar to μ_i , then $[\widehat{\nu}_1 \cdots \widehat{\nu}_r]$ is θ followed by conjugation by an element of B^* ; so it is an F -algebra isomorphism. ■

p. 297, second display: change the sign of the 1,2-entry, so the matrix becomes

$$\mu(a^2) = \begin{bmatrix} -1/2 & \sqrt{3}/2 \\ -\sqrt{3}/2 & -1/2 \end{bmatrix};$$

p. 297, line 9: replace “ $a = ba^2b^{-1}a^{-2}$ ” by “ $a = bab^{-1}a^{-1}$ ”

p. 305, third paragraph of proof, third line: replace $=$ by \cong , to make the equation $M \cong L_1^{n_1} \oplus \cdots \oplus L_r^{n_r}$

p. 309, line 7: insert a missing hat, so $\chi_i(c_j g_j)$ becomes $\widehat{\chi}_i(c_j g_j)$

p. 309, line 11: insert a missing hat, so it becomes

$$\frac{1}{n} \sum_{k=1}^r \widehat{\chi}_i(c_k g_k) \chi_j(g_{k'}) = (\chi_i | \chi_j).$$

p. 309, line 13: insert a missing hat, so it begins

$$\sum_{k=1}^r \chi_k(g_{i'}) \widehat{\chi}_k(c_j g_j) =$$

p. 309, line -7: replace “subring of” by “ring inside”

p. 309, line -5: replace “subring of” by “ring inside”

p. 319, line just before the first display: replace $r \in R$ by $r \in \mathcal{R}$

p. 335, line -9: insert “if $R^n \in \mathcal{C}$ ”, so it becomes

“ $R[x, x^{-1}]$ -Mod, if $R^n \in \mathcal{C}$, then $(M, \alpha) \cong (R^n, \cdot A)$ ”

p. 336, line -2: \overline{AB} should be $\overline{A} \overline{B}$

p. 339, line -9: replace $t \otimes (s \otimes \rho)$ by $t \otimes (s \otimes p)$

p. 345, line -4: replace $1 + arb$ by $I_n + arb$, so the matrix becomes

$$= \begin{bmatrix} I_n & 0 \\ -rb(I_n + arb)^{-1} & 1 \end{bmatrix}$$

p. 345, line -1: replace each n by $n + 1$; so it becomes

$$e_{1,n+1}(u)e_{n+1,1}(-u^{-1})e_{1,n+1}(u)e_{1,n+1}(-1)e_{n+1,1}(1)e_{1,n+1}(-1)$$

p. 347, second display: replace $P^{-1}(I_n + PabP^{-1})$ by $P^{-1}(I_n + PabP^{-1})P$

p. 352, line -13: replace “each $s_{n,n+1}$ is surjective” by
“each $s_{n,n+1}$ is injective”

p. 354, line -2 (display): E should be E'

p. 359, lines 6-10: “ $\varepsilon(x, y) = \cdots$ left to right.” should be:

“product

$$\varepsilon(x, y) = \begin{bmatrix} xy & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x & 0 \\ 0 & y \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 \\ (1-y) & 1 \end{bmatrix} \begin{bmatrix} 1 & x \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ (y-1)x^{-1} & 1 \end{bmatrix} \begin{bmatrix} 1 & -x \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & x(y-1)y^{-1} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ y(y-1)(xy-1)x^{-1} & 1 \end{bmatrix}$$

belongs to $E_{2n}(R, J)$.”

p. 360, line 15: replace ε by ϵ twice, so the phrase becomes

“ $B = v\epsilon_{j1}$ and $C = u\epsilon_{1i}$ ”

p. 365, line 11 should say $w \in J^{n \times 1}$

p. 365, line 12: A' should just be A , so the matrix becomes:

$$\begin{bmatrix} A & 0 \\ 0 & 1 \end{bmatrix}$$

p. 366, line -3: insert “by (11.1),” so the sentence begins

“For (iii), by (11.1),”

p. 367, line 1: replace “Since $E(R, J) \triangleleft GL(R)$,” with

“By (11.2), $E(R, J) \triangleleft GL(R)$,”

p. 382, line 9, omit second comma

p. 383, line 9: replace “Since $a' \neq 0$,” by

“If $a' = 0$, $J = R$ and $A' \in H$ by Step 3. Supposing $a' \neq 0$,”

p. 385, line -8: replace SL_2 by GL_2 and the period with a comma

p. 385, line -7: replace “So $\widehat{k}(N) = k \left(\begin{bmatrix} a & b \\ c & d \end{bmatrix} \right)$.” by

“with $ad - bc = \Delta \in GL_1(R, J)$. So $\widehat{k}(M) = k \left(\begin{bmatrix} a & b \\ c & d \end{bmatrix} \right)$.”

p. 387, line 10 should be: $= m\Delta - x(1 + me)c$.

p. 387, line 14, change the lower left entry, so the matrix is:

$$B = \begin{bmatrix} * & * \\ f & g \end{bmatrix} = \begin{bmatrix} * & * \\ m\Delta - x(1+me)c & d(1+me) \end{bmatrix}.$$

p. 387, line 15: delete “using $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \in SL_2(R, J)$,”

p. 387, line 18 should be: $= [d, m(\Delta - ad) - x(1+me)c][d, c]^{-1}$

p. 387, line 19 should be: $= [d, m\Delta - x(1+me)c][d, c]^{-1}$

p. 387, lines 21-25 should be:

$$\begin{aligned} k(B) &= [d(1+me), m\Delta - x(1+me)c] \\ &= [d, m\Delta - x(1+me)c][1+me, m\Delta - x(1+me)c] \\ &= [d, m\Delta - x(1+me)c][1+me, m\Delta] \\ &= [d, m\Delta - x(1+me)c][1, m\Delta] \\ &= [d, m\Delta - x(1+me)c]. \end{aligned}$$

p. 390, line -7: put a hat on k : $\widehat{k}(C^{-1}M^sC)^{-1}$

p. 391, line 3: $J^{2 \times 1}$ should be $J^{1 \times 2}$

p. 394, line 2: replace “then” by “then each factor”

p. 394, line 3: replace

$$\begin{pmatrix} b \\ a \end{pmatrix}_d \text{ by } \begin{pmatrix} b \\ P \end{pmatrix}_d^{v_P(aR)}$$

p. 395: In Lemma (11.34), change “Krull dimension at most 2” to “Krull dimension at most 1”

p. 396, line 12: $(P/pR)^*$ should be $(R/pR)^*$

p. 396, line 14: $1+cr$ should be $1+cR$

p. 397, line 4: “a real embedding” should be “no real embedding”

p. 407, line 8: change the subscript ik to jk , so the equation becomes:

$$x_{ji}((0, s)) = [x_{jk}((0, s)), x_{ki}((0, 1))] ,$$

pp. 409-410, replace by the following two pages:

Note: If $a, a', b, b' \in G$ and $\theta(a) = \theta(a')$ while $\theta(b) = \theta(b')$, then $u = a^{-1}a'$, $v = b^{-1}b'$ belong to $\ker(\theta)$, and hence to $Z(G)$. So $[a', b'] = [au, bv] = uu^{-1}vv^{-1}[a, b] = [a, b]$. This device is used frequently in the rest of the proof.

Now if $k \neq i$ and $k \neq j$,

$$y_{ij}(r) = [y_{ik}(r), y_{kj}(1)] = [y'_{ik}(r), y'_{kj}(1)] = y'_{ij}(r).$$

So if f exists, it is unique. To prove f exists, we employ a few commutator identities. The last one is the **Jacobi identity**:

(12.12) Lemma. *Suppose G is a group and $a, b, c \in G$.*

- (i) $[ab, c] = [a, [b, c]] [b, c] [a, c]$.
- (ii) *If $[a, b]$ and $[a, c]$ are in $Z(G)$, then a commutes with $[b, c]$.*
- (iii) *If $[G, G]$ is abelian, $[a, [b, c]] [b, [c, a]] [c, [a, b]] = 1$.*

Proof. For (i), expand and simplify. For (ii), $a[b, c]a^{-1} = [aba^{-1}, aca^{-1}] = [[a, b]b, [a, c]c] = [b, c]$. Using (i) the expression in (iii) simplifies to the product $[ab, c] [ca, b] [bc, a] = 1$. ■

Now, to show f exists, we first make a blind choice of one element $z_{ij}(r)$ from each set $\theta^{-1}(e_{ij}(r))$. One of the Steinberg relations does hold: If $i \neq \ell$, $j \neq k$ and $m \notin \{i, j, k, \ell\}$, then by (12.12) (ii),

$$[z_{ij}(r), z_{k\ell}(s)] = [z_{ij}(r), [z_{km}(s), z_{m\ell}(1)]] = 1.$$

But the other Steinberg relations need not hold for the $z_{ij}(r)$. If there is a choice of $y_{ij}(r) \in \theta^{-1}(e_{ij}(r))$ satisfying the Steinberg relations, we would have $y_{ij}(r) = [y_{im}(r), y_{mj}(1)] = [z_{im}(r), z_{mj}(1)]$ for each m not equal to i or j . So we define

$$y_{ij}^m(r) = [z_{im}(r), z_{mj}(1)] \in \theta^{-1}(e_{ij}(r)).$$

It only remains to prove $y_{ij}^m(r)$ is independent of m and obeys the Steinberg relations. Suppose i, j, m , and n are four different positive integers, and $r \in R$. Taking $u = z_{in}(r)$, $v = z_{nm}(s)$, and $w = z_{mj}(t)$, and G the group generated by u, v , and w , we see by the Steinberg relation verified above that $[u, w] = 1$. So $[G, G]$ is generated by the elements $gxg^{-1} = [g, x]x$ where $g \in G$ and x is $[u, v]$ or $[v, w]$. Now $\theta([u, v]) = e_{im}(rs)$ and $\theta([v, w]) = e_{nj}(st)$; so $[u, v]$ commutes with $[v, w]$. Similarly $[u, v]$ commutes with u, u^{-1}, v , and v^{-1} , and for $i = 1$ or -1 , $[w^i, [u, v]]$ commutes with u, v , and w , so lies in $Z(G)$. Taking $c = [u, v]$, $a \in G$, and $b = u^i, v^i$, or w^i in (12.12) (i),

$$[ab, [u, v]] = [b, [u, v]] [a, [u, v]].$$

Repeating this, $[g, [u, v]] \in Z(G)$ for all $g \in G$. Similarly each $[g, [v, w]] \in Z(G)$. So $[G, G]$ is abelian. Replacing a, b, c by u, v, w in the Jacobi identity, and recalling that $[u, w] = 1$, we get

$$(12.13) \quad [u, [v, w]] = [[u, v], w].$$

With $s = t = 1$, this becomes

$$\begin{aligned} y_{ij}^m(r) &= [[z_{in}(r), z_{nm}(1)], z_{mj}(1)] \\ &= [z_{in}(r), [z_{nm}(1), z_{mj}(1)]] = y_{ij}^n(r). \end{aligned}$$

So $y_{ij}^m(r)$ is independent of m , and we rename it $y_{ij}(r)$. Now suppose i, j, k, ℓ are positive integers with $i \neq j, k \neq \ell, i \neq \ell$, and $j \neq k$. Replacing our original choice of $z_{ij}(r)$ by $y_{ij}(r)$, we have seen that

$$[y_{ij}(r), y_{k\ell}(s)] = 1.$$

By definition of $y_{ij}^m(r)$, we also know

$$[y_{im}(r), y_{mj}(1)] = y_{ij}(r)$$

whenever $m \neq i$ and $m \neq j$. Suppose i, j, m , and n are four different positive integers. Take u, v, w , as above with $t = 1$. Then by (12.13),

$$\begin{aligned} [y_{in}(r), y_{nj}(s)] &= [y_{in}(r), [y_{nm}(s), y_{mj}(1)]] \\ &= [[y_{in}(r), y_{nm}(s)], y_{mj}(1)] \\ &= [y_{im}(rs), y_{mj}(1)] \\ &= y_{ij}(rs). \end{aligned}$$

For the remaining relation, if i, j, m are three different positive integers,

$$\begin{aligned} y_{ij}(r+s) &= y_{ij}(s+r) \\ &= [y_{im}(s+r), y_{mj}(1)] \\ &= [y_{im}(s)y_{im}(r), y_{mj}(1)] \\ &= [y_{im}(r), y_{mj}(1)] [y_{im}(s), y_{mj}(1)] \\ &= y_{ij}(r)y_{ij}(s), \end{aligned}$$

where the penultimate equality is (12.12)(i) with $a = y_{im}(s), b = y_{im}(r)$, and $c = y_{mj}(1)$, so that $[a, [b, c]] = [y_{im}(s), y_{ij}(r)] = 1$. This completes the proof that ϕ is a universal central extension. \blacksquare

p. 414, line 16: replace $GL(R)$ by $GL(R)$

p. 414, line 17: insert $= P_\sigma$, so the line becomes

$$P(\sigma) = P_\sigma = \sum_{i=1}^n \epsilon_{\sigma(i)i} ,$$

p. 414, line 19: insert a missing τ subscript and delete a composition sign in another subscript; so it becomes:

$$P(\sigma)P(\tau) = \sum \epsilon_{\sigma(i)i} \sum \epsilon_{\tau(j)j} = \sum \epsilon_{\sigma\tau(j)j} = P(\sigma \circ \tau) .$$

p. 414, line -2: replace i by j in a subscript, so the sum becomes $\sum d_{\sigma(j)} \epsilon_{jj}$

p. 416, line 2: replace $\psi(w)$ by $\phi(w)$

p. 416, (12.19)(i): the right side should be $x_{ij}(b)$, so it becomes

$$w_{k\ell}(a)x_{ij}(b)w_{k\ell}(a)^{-1} = x_{ij}(b)$$

p. 417, line 7: insert a minus sign in the next to last factor, making it $x_{jk}(-a^{-1})$

p. 418, line -3: should be

$$\phi([h, h']) = [\phi(h), \phi(h')] = 1 ,$$

p. 420, line 3: remove the comma in the subscript, so it becomes $\{a, b\}_{ij}$

p. 420, line 7: replace $[a, b]_{12}$ by $\{a, b\}_{12}$

p. 421, line 9: the first $=$ should be \equiv , making it

$$w_{ji}(1) \equiv w_{ij}(1)^{-1} = w_{ij}(-1) \equiv w_{ij}(1) .$$

p. 427, line -6: replace \neq by “and” and delete the word “permutation”, so it becomes:

“For positive integers i and j , let $P(i, j)$ denote the matrix”

p. 427, line -3: replace “Prove that” by “If $i \neq j$, show”

p. 428, line 1: delete “that”

p. 428, exercise 3, line 2: replace w by W , so the equation becomes

$$\phi(W) = (P(R)D(R)) \cap E(R)$$

p. 428, exercise 4, display: delete the (before the last h_{ij}

- p. 431, last paragraph: change the first " to "
- p. 445, first display: the upper left corner should be \overline{P}_1
- p. 449, middle of page: replace "Lemma (13.17)" by "Lemma (13.18)"
- p. 455, Exercise 13: replace the citation of "Exercise 9" by "Exercise 12"
- p. 457, in the line after the first display: replace $\overline{R} = R/J$ by $\overline{R} = R/\mathfrak{J}$
- p. 463, line 5: replace $g_1(x)(B)$ by $g_1(x)B$
- p. 463, line 15: replace (θ_1, θ_2) by (θ_1, θ_1)
- p. 464, line -4: replace θ_i by θ_2
- p. 466, line 11: replace the last term (R^n, \overline{A}^n, C) by (R^n, \overline{A}^n, B)
- p. 473, just before (13.35): replace "Exercises 3 and 6" by "Exercise 3"
- p. 475, in (13.36)(ii): the last equation was truncated and should be $r = \phi(p^{u+v})/\phi(p^u)$
- p. 476, line -11: replace \overline{M}_i by \overline{M}_i , so the product becomes $\overline{R} \times \cdots \times \overline{M}_i \times \cdots \times \overline{R}$
- p. 481, Hint to Exercise 5: replace (13.19) by (13.20), and (13.27) by (13.30)
- p. 482, third display: replace $[A/A^n s\alpha]$ by $[A^n/A^n s\alpha]$, making the equation

$$d_n(\alpha) = [A^n/A^n s\alpha] - [A^n/A^n s] .$$

- p. 484, in the middle of the page: add) to make the citation (see Bass [68, XII, §1])
- p. 485, line 3: replace " $\mathcal{F} : \mathbf{Ring}$ to $\mathcal{A}\mathfrak{b}$ " by " $F : \mathbf{Ring} \rightarrow \mathcal{A}\mathfrak{b}$ "
- p. 487, Exercise 6: insert the phrase "acting through injections on R and", to make it begin:
- "6. If R is a commutative ring and S is a submonoid of (R, \cdot) acting through injections on R and meeting all but finitely many maximal ideals of R ,"
- p. 489, second paragraph, line 1: replace " R -module" by " R -algebra"
- p. 497, in (14.9): replace F by \mathcal{F}
- p. 497, just before the last paragraph: replace Chapter 6 by (5.9)
- p. 506, Example (14.22), line 6: replace "As in (13.18)" by "As on p. 74"
- p. 516, next-to-last line before 14D. Exercises: replace " $I \oplus J \cong R \oplus IJ$ " by " $I \otimes_R J \cong IJ$ "
- p. 529, in the proof of (14.47): replace the three sentences

“The kernel B of σ is generated by elements $\ell(a_1) \cdots \ell(a_r) \ell(-a_{r+1}) \cdots \ell(-a_n)$, where a_1, \dots, a_n are positive and $1 \leq r \leq n$. Each $\ell(-a_j) = \ell(-1) + \ell(a_j)$; so this generator is a sum of terms $\pm \ell(a_1) \cdots \ell(a_r) \ell(-1)^s \ell(b_1) \cdots \ell(b_{n-r-s})$, where each b_i is positive. And $\ell(-1)^s \ell(b_1) = \ell(b_1)^{s+1}$.”

by

“Each $\ell(-a) = \ell(-1) + \ell(a)$; so $K_n^M(\mathbb{R})$ is generated by elements $\ell(-1)^s \ell(b_1) \cdots \ell(b_{n-s})$ with $0 \leq s \leq n$ and all b_i positive. If $s < n$, this term equals $\ell(b_1)^{s+1} \cdots \ell(b_{n-s})$. Terms with $s = n$ are $\ell(-1)^n$, and these cancel in pairs.”

p. 534, line 5: replace “has large enough rank” by
“has nonzero exterior powers”

p. 547, just before the fourth display: replace “ b ” by “ $f(a)$ ”, so the line becomes:

“So multiplication by a on A and multiplication by $f(a)$ on B are represented by”

p. 558, the first line after diagram (14.70): replace $K_2(R, J)$ by $K_2(R/J)$

p. 560, line -5: replace “For each integer n ,” by “Now”

p. 560, the next to the last display should be replaced by

$$\ell\left(1 - \frac{1}{t}\right)\ell(1-t) = 0 = \ell(t)\ell\left(1 - \frac{1}{t}\right).$$

p. 576, last display: omit the first $+J$, making it

$$\|\widehat{x}_j - (\{x_i\} + J)\|^\wedge = \lim_i \|x_j - x_i\| < \varepsilon,$$

p. 602, replace lines 1 and 2 by:

“ $\mathcal{O}_v[x]$, $\bar{t}\alpha + \bar{u}\beta = 1$. Lift α, β to $a_0, b_0 \in \mathcal{O}_v[x]$ of the same degree. So $ta_0 + ub_0 - 1$ and $f - a_0b_0$ lie in $\pi\mathcal{O}_v[x]$. If $f = a_0b_0$, we are done.”

p. 602, line 21: replace “each nonzero” by “the leading”, and replace t_n by s_n , so it becomes:

“cient of a_0 times the leading coefficient of s_n is not in $c\mathcal{O}_v$. So $\deg(s_n) \leq D-d$.”

p. 603, line 7: replace $\pi^N \mathcal{O}_v$ by $\pi^N \mathcal{O}_v[x]$

p. 603, line 8: replace $\pi^N \mathcal{O}_v$ by $\pi^N \mathcal{O}_v[x]$

p. 616, line 5: replace “ $Z(X)$ and $Z(Y)$ ” by “ $Z(A)$ and $Z(B)$ ”

p. 619-620: replace p. 619, line -7 to p. 620, line 3 by:

“Since $M_n(F) \otimes_F M_m(F) \cong M_{mn}(F)$, the elements of \mathcal{F} each have the form $[M_s(F)][M_r(F)]^{-1}$. Thus $\overline{[A]} = \overline{[B]}$ in $Br(F)$ if and only if

$$\begin{aligned} [A] [B]^{-1} &= [M_s(F)] [M_r(F)]^{-1}; \text{ so} \\ [M_r(A)] &= [A] [M_r(F)] = [B] [M_s(F)] = [M_s(B)], \end{aligned}$$

for positive integers r, s . Combining this with the criterion for equality in $BrG(F)$, we find $\overline{[A]} = \overline{[B]}$ in $Br(F)$ if and only if

$$M_u(A) \cong M_v(B)$$

as F -algebras, for some positive integers u and v . When the latter condition holds, we say A is **similar** to B , or write $A \sim B$. Evidently similarity is an equivalence relation among the objects of $\mathcal{A}_3(F)$.”

p. 620, line 18: replace “**algebra of F** ” by “**algebra of A** ”

p. 622, Exercise 5, second display: replace $Z_A(A') \otimes_B Z_B(B')$ by $Z_A(A') \otimes_F Z_B(B')$

p. 622, Exercise 6, second display: replace A by B twice, so it becomes:

$$I \supseteq (I^* \otimes 1)(1 \otimes B) = I^* \otimes_F B.$$

p. 622, Exercise 6, second sentence after second display: replace “Then $a_j \otimes 1 = (f_j \otimes 1)(c)$ ” by “Then $a_j \otimes 1 = (1 \otimes f_j)(c)$ ”

p. 634, line 1: replace $\delta_1(c)(g)$ by $\delta_1(c)(g, h)$

p. 638, proof of Lemma (16.26): in the first display line, replace n by r , so the right side becomes:

$$\frac{p_r(t)}{p_i(t)p_{r-i}(t)}$$

p. 638, proof of Lemma (16.26): in the third display line, replace the exponent $r - 1$ by $r - i$, making the second factor:

$$\left[t^i \left(\frac{1}{t^i - 1} \right) + \frac{1}{t^{r-i} - 1} \right]$$

p. 642, line 24: replace θ by $R_{n,F}$ making the sentence:

“In 1976, Tate [76] proved $R_{n,F}$ is injective for local and global fields.”