Pulsons, peakons and solutions of 1+1 evolutionary PDEs that act like billiards

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Abstract

We discuss novel wave structures present in the class of 1+1 dimensional evolutionary PDEs

$$m_t + um_x + b\,u_x m = 0\,, \quad u = g * m\,, \quad \lim_{|x|\to\infty} u, m = 0\,. \qquad (1)$$

Here $u = g*m$ is convolution of $m(x,t)$ with a spatial filter (or Green's function) $g(x)$, and b is a constant. When g is even and bounded, these PDEs yield propagating wave solutions for $u(x,t)$, with shape g. These are the *pulsons*, and they collide elastically like billiards. For some choices of the function g and the constant b, the solution develops verticality in finite time at inflection points of negative slope. The choice $g(x) = e^{-|x|}$ avoids developing verticality, by avoiding inflection points and introducing a jump in slope at the peak. These are the *peakons*. For b=2 and b=3, the initial value problem for the peakon equation can be completely solved by using methods from soliton theory.

I will mention how the above PDEs arise in the 1D compressible limit of a new 3D turbulence model called the Navier-Stokes$-\alpha$ model.