

Math 120 Merit Workshop  
Worksheet 10

1. You are traveling in your car in such a way that your position as a function of time is give by the formula  $s(t) = t^2 - 8t + 18$ .

a) Find your velocity at  $t = 2$ .

b) At time  $t = 4$  you are at a stop sign. You claim that you stopped but a traffic cop claims that you didn't. Who is right?

2. Let's revisit Particle Man. As he is traveling along his path one day, he runs into a rather shapely Particle Woman. They chat for a few minutes and (being the young and healthy particle that he is), Particle Man decides to see if she would like to meet him later for a cup of coffee. She is the coy type and gives him this cryptic answer: "I'll be at the coordinate  $(A, B)$  in two hours." When Particle Man asks for a more specific meeting spot she replies: "Well,  $A$  is the slope of the tangent line to  $f(x) = -2x^2 + 4$  at the point  $(1, 2)$  and  $B$  is the slope of the tangent line to  $g(x) = 70x - x^2$  at the points  $(3, 201)$ ." As she smiles sweetly and moves along her way, Particle Man stares dejectedly into space because he knows that he could never decipher Particle Woman's confusing riddle. Take pity on the poor guy and find out where she will be in two hours!

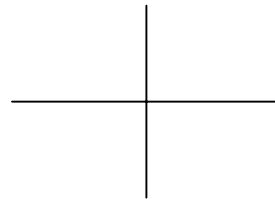
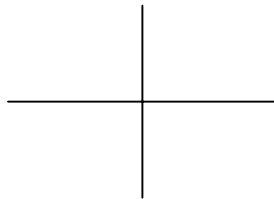
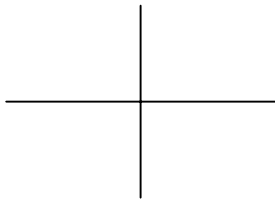
3. After Particle Man deciphered Particle Woman's riddle (with your help of course), he cleaned himself up and headed to the appropriate coordinate. Particle Woman was already there when he arrived, and he walked up to her ready to pour on the charm. Unfortunately, Particle Woman's boyfriend, The Mighty Molecule Man, showed up just then and was furious to see someone hitting on his woman. In his anger, Molecule Man shoves Particle Man into a cannon and shoots him vertically into the air. Particle Man's height as a function of time is given by the function  $s(t) = 80t - 16t^2$ .

a) Write an equation describing his velocity as a function of time.

b) When does he reach his maximum height and what is that height?

c) When does he hit the ground and what is his velocity when he hits?

4. For each function, sketch the graph of its derivative



5. The temperature,  $T$ , in degrees Fahrenheit, of a cold yam placed in a hot oven is given by  $T = f(t)$ , where  $t$  is the time in minutes since the yam was put in the oven.

(a) What is the sign of  $f'(t)$ ? Why?

(b) What are the units of  $f'(20)$ ? What is the practical meaning of the statement  $f'(20) = 2$ ?

6. What does it mean for a function to be *differentiable*? What might the graph of  $f(x)$  look like if it is NOT differentiable at  $x = a$ ?

7. Suppose a function  $f$  is continuous at  $x = a$ . Does this mean the function is differentiable at  $x = a$ ?
8. Prove each of the following using the definitions of even and odd functions
- The derivative of an even function is an odd function.
  - The derivative of an odd function is an even function.

*Some review for Quiz #2*

9. a) Write down the intermediate value theorem.  
b) Explain the intermediate value theorem to someone. If you don't yet understand it, have someone explain it to you until you are able to explain it to someone else.  
c) Use the intermediate value theorem to show there is a root of  $\ln x = e^{-x}$  on the interval  $(1, 2)$ .
10. Suppose that a function is continuous on  $[0, 1]$  except at  $\frac{1}{2}$  and that  $f(0) = 1$  and  $f(1) = 3$ . Sketch two possible graphs of  $f$ , one showing that  $f$  might not satisfy the conclusion of the intermediate value theorem and one showing that  $f$  might still satisfy the conclusion even though it doesn't satisfy the hypothesis.
11. Find the vertical and horizontal asymptotes of  $f(x) = \frac{1-x}{(2-x)^2}$  and  $g(x) = \frac{x^2+4}{2x^2-2}$ .

12. Find the points where the following functions are discontinuous. For each point tell whether the discontinuity is removable:

a)  $f(x) = \frac{1}{1-|x|}$

b)  $f(x) = \frac{3x^2 - 7x - 20}{x - 4}$

13. Find the constant  $c$  that makes  $f(x)$  continuous at  $x = 3$ :

$$f(x) = \begin{cases} cx + 1 & \text{if } x \leq 3 \\ cx^2 - 1 & \text{if } x > 3 \end{cases}$$

14. Find the limit:  $\lim_{t \rightarrow \infty} \frac{7t^3 + 4t}{2t^3 - t^2 + 3}$

15. Find  $\lim_{x \rightarrow \infty} h(x)$  and  $\lim_{x \rightarrow -\infty} h(x)$ :

$$h(x) = x^3(x-2)(1-x)$$